ep 7-2: 5 with sign change:

1st order linear nonhomogeneous DE system

$$\underline{x}' = \underline{A}\,\underline{x} + \underline{F} \qquad \underline{A} = \begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} \qquad \underline{F} = \begin{bmatrix} 3e^t \\ -t^2 \end{bmatrix}$$

HAND WORK

eigenvalues $\lambda = 6, -3$

eigenvectors $\underline{B} = \begin{bmatrix} 1 & -4 \\ 1 & 5 \end{bmatrix}$ ($\underline{b_1}$ $\underline{b_2}$) $\qquad \underline{B}^{-1} = \frac{1}{9}\begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix} \qquad \underline{B}^{-1}\underline{F} = \frac{1}{9}\begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3e^t \\ -t^2 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 15e^t - 4t^2 \\ -3e^t - t^2 \end{bmatrix}$

$\underline{x} = \underline{B}\,\underline{y}$

$\underline{y} = \underline{B}^{-1}\underline{x}$

$$A_B = B^{-1}AB = \begin{bmatrix} 6 & 0 \\ 0 & -3 \end{bmatrix}$$

$$B^{-1}\left[(\underline{B}\underline{y})' = A(\underline{B}\underline{y}) + \underline{F}\right]$$

$$\underline{y}' = \underline{A_B}\,\underline{y} + \underline{B}^{-1}\underline{F} \qquad \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} \frac{15}{9}e^t - \frac{4}{9}t^2 \\ -\frac{1}{3}e^t - \frac{1}{9}t^2 \end{bmatrix}$$

particular soln

$$y_1' = 6y_1 + \tfrac{15}{9}e^t - \tfrac{4}{9}t^2 \rightarrow y_1' - 6y_1 = \tfrac{15}{9}e^t - \tfrac{4}{9}t^2 \rightarrow y_{1p} = c_3e^t + c_4 + c_5t + c_6t^2$$

$$\underbrace{y_2' = -3y_2}\; -\tfrac{1}{3}e^t - \tfrac{1}{9}t^2 \rightarrow y_2' + 3y_2 = -\tfrac{1}{3}e^t - \tfrac{1}{9}t^2 \rightarrow y_{2p} = c_7e^t + c_8 + c_9t + c_{10}t^2$$

$y_{1h} = c_1e^{6t}$

$y_{2h} = c_2e^{-3t}$

homogeneous soln

backsub:

$$y_{1p}' - 6y_{1p} = (c_3e^t + c_5 + 2c_6t) - 6(c_3e^t + c_4 + c_5t + c_6t^2)$$
$$= \underbrace{-5c_3}_{15/9}e^t + \underbrace{(c_5 - 6c_4)}_{0} + \underbrace{(2c_6 - 6c_5)}_{0}t \underbrace{-6c_6}_{-\frac{4}{9}}t^2 = \tfrac{15}{9}e^t - \tfrac{4}{9}t^2$$

$$y_{2p}' + 3y_{2p} = (c_7e^t + c_9 + 2c_{10}t) + 3(c_7e^t + c_8 + c_9t + c_{10}t^2)$$
$$= \underbrace{4c_7}_{-1/3}e^t + \underbrace{(c_9 + 3c_8)}_{0} + \underbrace{(2c_{10} + 3c_9)}_{0}t + \underbrace{3c_{10}}_{-1/9}t^2 = -\tfrac{1}{3}e^t - \tfrac{1}{9}t^2$$

$-5c_3 = 15/9 \rightarrow c_3 = -1/3$

$-6c_4 + c_5 = 0 \longrightarrow c_4 = \frac{1}{6}c_5 = \frac{1}{3\cdot 81}$

$-6c_5 + 2c_6 = 0 \longrightarrow c_5 = \frac{1}{3}c_6 = \frac{2}{81}$

$-6c_6 = -4/9 \rightarrow c_6 = 2/27$

$4c_7 = -1/3 \rightarrow c_7 = -\frac{1}{12}$

$+3c_8 + c_9 = 0 \longrightarrow c_8 = -\frac{1}{3}c_9 = \frac{-2}{3\cdot 81}$

$3c_9 + 2c_{10} = 0 \longrightarrow c_9 = -\frac{2}{3}c_{10} = \frac{2}{81}$

$3c_{10} = -1/9 \rightarrow c_{10} = -1/27$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} c_1e^{6t} & -\frac{1}{3}e^t + \frac{1}{3\cdot 81} + \frac{2}{81}t + \frac{2}{27}t^2 \\ c_2e^{-3t} & -\frac{1}{12}e^t - \frac{2}{3\cdot 81} + \frac{2}{81}t - \frac{1}{27}t^2 \end{bmatrix}$$

$$= \begin{bmatrix} c_1e^{6t} - 4c_2e^{-3t} & + \left(-\frac{1}{3} + \frac{1}{3}\right)e^t + \left(\frac{1+8}{3\cdot 81}\right) + \left(\frac{2}{81}(1-4)\right)t + \frac{1}{27}(2+4)t^2 \\ c_1e^{6t} + 5c_2e^{-3t} & \left(-\frac{1}{3} - \frac{5}{12}\right)e^t + \left(\frac{1-10}{3\cdot 81}\right) + \left(\frac{2}{81}(1+5)\right)t \quad \frac{1}{27}(2-5)t^2 \end{bmatrix}$$

$$= \begin{bmatrix} c_1e^{6t} - 4c_2e^{-3t} & + 0e^t & + \frac{1}{27} & -\frac{2}{27}t & + \frac{2}{9}t^2 \\ c_1e^{6t} + 5c_2e^{-3t} & -\frac{3}{4}e^t & -\frac{1}{27} & +\frac{4}{27}t & -\frac{1}{9}t^2 \end{bmatrix}$$

✓ (agrees with maple $c_1 = -C2,\ 5c_2 = -C1$)

we could have directly backsubstituted $x_{1p} = a_1e^t + a_2 + a_3t + a_4t^2$, $x_{2p} = a_5e^t + a_6 + a_7t + a_8t^2$

to find the particular soln without the additional transformation of coords