Definite integral functions as Antiderivatives

Consider IVP: $\frac{dy}{dx} = f(x)$, $y(a) = y_0$

Then it has the solution:

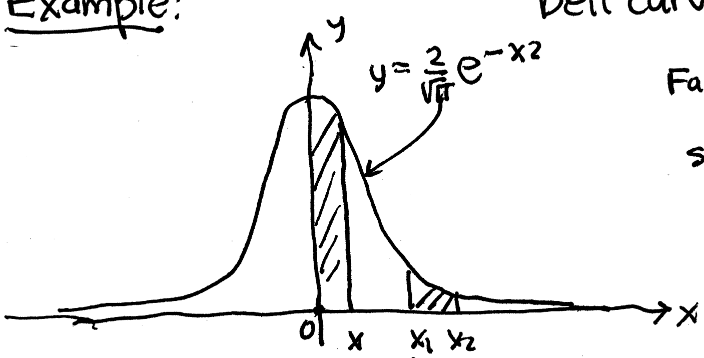
$$y = \int f(x)\,dx = \int_a^x f(t)\,dt + y_0$$

dummy variable

Pf: Let $F(x) = \int_a^x f(t)\,dt \to F'(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

$F(a) = \int_a^a f(t)\,dt = 0 \to y(a) = \underbrace{F(a)}_{0} + y_0 = y_0$

Example: "bell curve" (normal distribution)



Fact: $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$ (multivariable calculus)

so $\int_{-\infty}^{\infty} \underbrace{\frac{2}{\sqrt{\pi}} e^{-t^2}}_{\geq 0}\,dt = 1 \to$ probability distribution

area

$$\begin{cases} F(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-t^2}\,dt \equiv \mathrm{erf}(x) \\ F(0) = 0 \end{cases}$$

area = Probability of value between x_1 and x_2
$= F(x_2) - F(x_1)$

$F'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$ i.e. solution of $\frac{dy}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$, $y(0) = 0$

$= \mathrm{erf}'(x)$

so $\left(\underbrace{\frac{\sqrt{\pi}}{2}\mathrm{erf}(x)}_{\text{antiderivative of } e^{-x^2}}\right)' = e^{-x^2}$