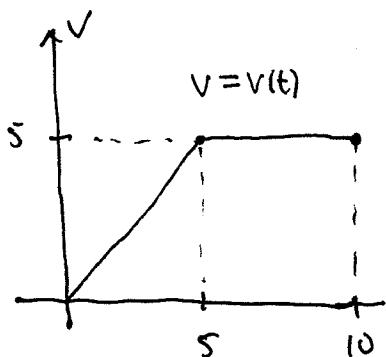


E&P 3 1.2.19

A particle starts at the origin and travels along the x-axis with the velocity function $v(t)$ shown in the graph. Sketch the graph of the resulting position function $x(t)$ for $0 \leq t \leq 10$.



solution: $v = \begin{cases} t, & 0 \leq t \leq 5 \\ 5, & 5 \leq t \leq 10 \end{cases}$

For $0 \leq t \leq 5$: $\begin{cases} \frac{dx}{dt} = t \\ x(0) = 0 \end{cases} \rightarrow x = \int t dt = \frac{t^2}{2} + C_1 \rightarrow x = \frac{1}{2}t^2$

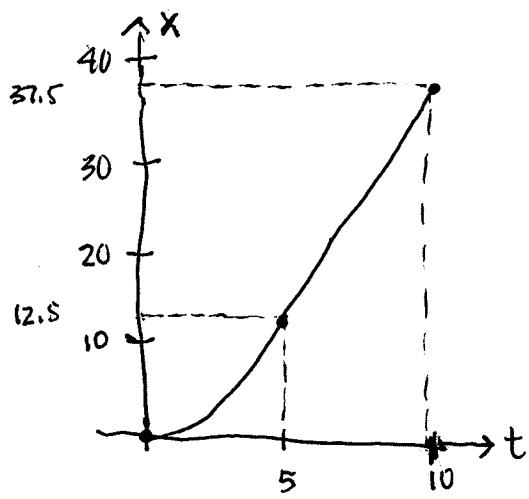
$$0 = x(0) = 0 + C_1 \rightarrow C_1 = 0$$

$$x(5) = \frac{1}{2} \cdot 5^2 = \frac{25}{2} = 12.5$$

For $5 \leq t \leq 10$: $\begin{cases} \frac{dx}{dt} = 5 \\ x(5) = \frac{25}{2} \end{cases} \rightarrow x = \int 5 dt = 5t + C_2$

$$\frac{25}{2} = x(5) = 5(5) + C_2 \rightarrow C_2 = -\frac{25}{2}$$

$$x(10) = 50 - \frac{25}{2} = \frac{75}{2} = 37.5$$



This example is useful since it motivates IVP problems not starting at $t=0$.

For piecewise defined functions $f(x)$ in $\frac{dy}{dx} = f(x)$, we need to use the final value of each interval as the initial value for the next one.

This will return in Chapter 2 with air resistance where an integral formula leads to a piecewise defined velocity function.