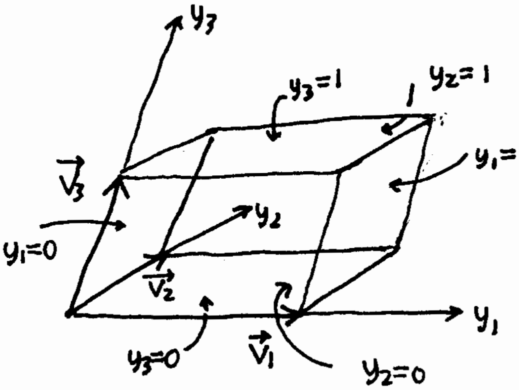
Nonstandard Coordinates in $\mathbb{R}^3$

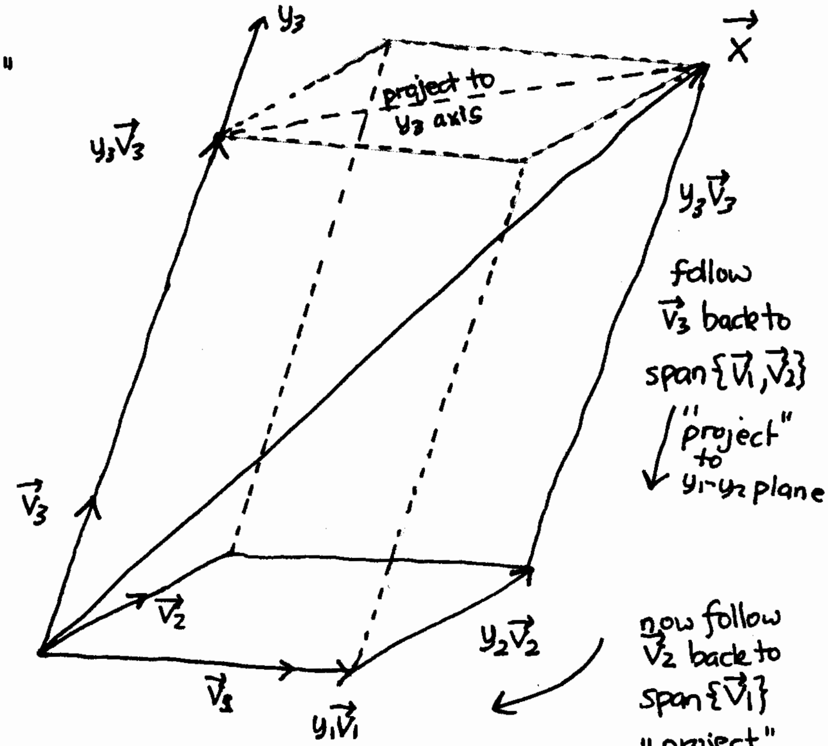
basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of $\mathbb{R}^3$ "new basis"
= any 3 LI vectors



y_i coord lines are parallel to vector $\vec{v}_i$

y_i coord planes are parallel to planes of remaining 2 vectors

This parallelopiped above represents all points $\vec{x}$ for which:
$0 \le y_1 \le 1, \quad 0 \le y_2 \le 1, \quad 0 \le y_3 \le 1.$



$$\vec{x} = ((y_1\vec{v}_1) + y_2\vec{v}_2) + y_3\vec{v}_3$$

first $\leftarrow$ to $\leftarrow$ last

■ GEOMETRICAL DETERMINATION OF NEW COORDINATES OF A VECTOR $\vec{x}$

■ MATRIX DETERMINATION OF NEW COORDINATES OF A VECTOR $\vec{x}$

Augment basis vectors into columns of a matrix (basis changing matrix)

$$B = \text{augment}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{array}{c} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\ \begin{matrix} \uparrow\quad & \uparrow\quad & \uparrow \\ \vec{v}_1\quad & \vec{v}_2\quad & \vec{v}_3 \end{matrix} \end{array}$$

each column consists of the old (standard) coordinates of a new basis vector

Now express any vector (standard coordinates) in terms of the new basis:

$$\underset{\text{"old coords"}}{\vec{x}} = y_1\vec{v}_1 + y_2\vec{v}_2 + y_3\vec{v}_3 = B\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = B\underset{\text{"new coords"}}{\vec{y}}$$

Solve $\vec{x} = B\vec{y}$ to find new coords $\vec{y}$ of $\vec{x}$.

LI of new basis vectors guarantees that if $\vec{x} = 0$ then $\vec{y} = 0$ is only solution, so $\det(B) \neq 0$ and B^{-1} exists. Solution equivalent to: $\underset{\text{"new coords"}}{\vec{y}} = B^{-1}\underset{\text{"old coords"}}{\vec{x}}$
unique!