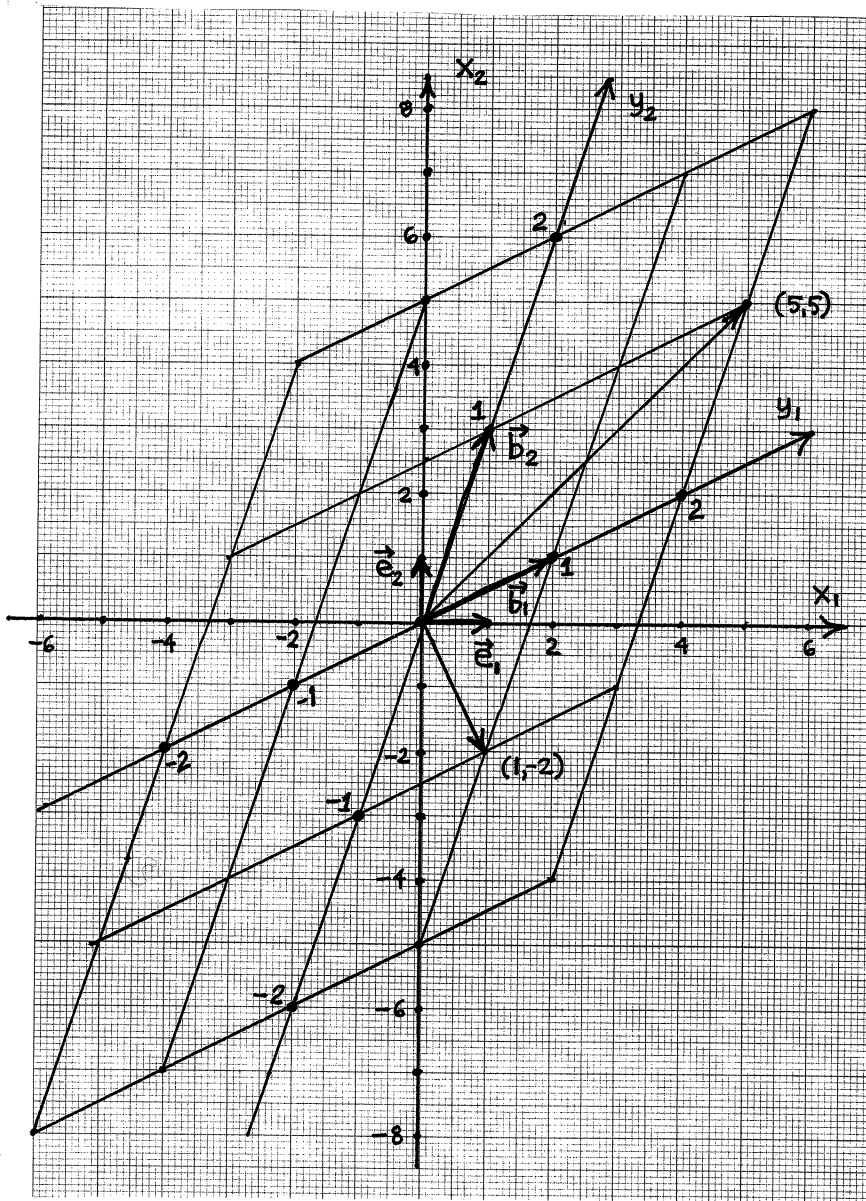


Nonstandard coordinates on \mathbb{R}^2



old basis:
 $\vec{e}_1 = \langle 1, 0 \rangle$, $\vec{e}_2 = \langle 0, 1 \rangle$

new basis:
 $\vec{b}_1 = \langle 2, 1 \rangle$, $\vec{b}_2 = \langle 1, 3 \rangle$

basis changing matrix:

$$B = \text{augment}(\vec{b}_1, \vec{b}_2) = \langle \vec{b}_1, \vec{b}_2 \rangle$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \begin{array}{l} \text{cols} = \\ \text{old coords of} \\ \text{new basis vectors} \end{array}$$

inverse:

$$B^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{bmatrix}$$

cols = new coords of
old basis vectors

2 example vectors:

$$\langle 5, 5 \rangle = 5\vec{e}_1 + 5\vec{e}_2$$

$$= 2\vec{b}_1 + 1\vec{b}_2$$

$$x_1 = 5, x_2 = 5$$

$$y_1 = 2, y_2 = 1$$

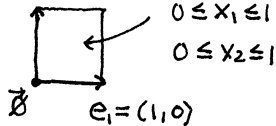
$$\langle 1, -2 \rangle = 1\vec{e}_1 - 2\vec{e}_2$$

$$= 1\vec{b}_1 - 1\vec{b}_2$$

$$x_1 = 1, x_2 = -2$$

$$y_1 = 1, y_2 = -1$$

$$\vec{e}_2 = (0, 1)$$



$$0 \leq x_1 \leq 1$$

$$0 \leq x_2 \leq 1$$

$$\vec{e}_1 = (1, 0)$$

The graph paper has the unit coordinate grid for the standard cartesian coordinates $\{x_1, x_2\}$ marked by bold line 1cm tickmarks (refined to 1mm tenth tickmarks).

The unit coordinate rectangle at the origin tiles the plane to make this grid.

The new coordinate grid is a tiling of the plane by the unit coordinate "rectangle" $0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1$ (actually a parallelogram) at the origin. The region shown is $-2 \leq y_1 \leq 2, -2 \leq y_2 \leq 2$.

The relationship between the two coordinate systems

$$\vec{X} = \langle x_1, x_2 \rangle = x_1 \vec{e}_1 + x_2 \vec{e}_2 = y_1 \vec{b}_1 + y_2 \vec{b}_2 = y_1 \langle 2, 1 \rangle + y_2 \langle 1, 3 \rangle$$

$$\text{or } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = y_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \iff \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{or } \begin{array}{cc} X & = & B Y \\ \text{"old"} & & \text{"new"} \end{array} \iff \begin{array}{cc} Y & = & B^{-1} X \\ \text{"new"} & & \text{"old"} \end{array}$$

$$\text{or } \begin{array}{l} x_1 = 2y_1 + y_2 \\ x_2 = y_1 + 3y_2 \end{array}$$

linear change
of coordinates

$$y_1 = (3x_1 - x_2) / 5$$

$$y_2 = (-x_1 + 2x_2) / 5$$