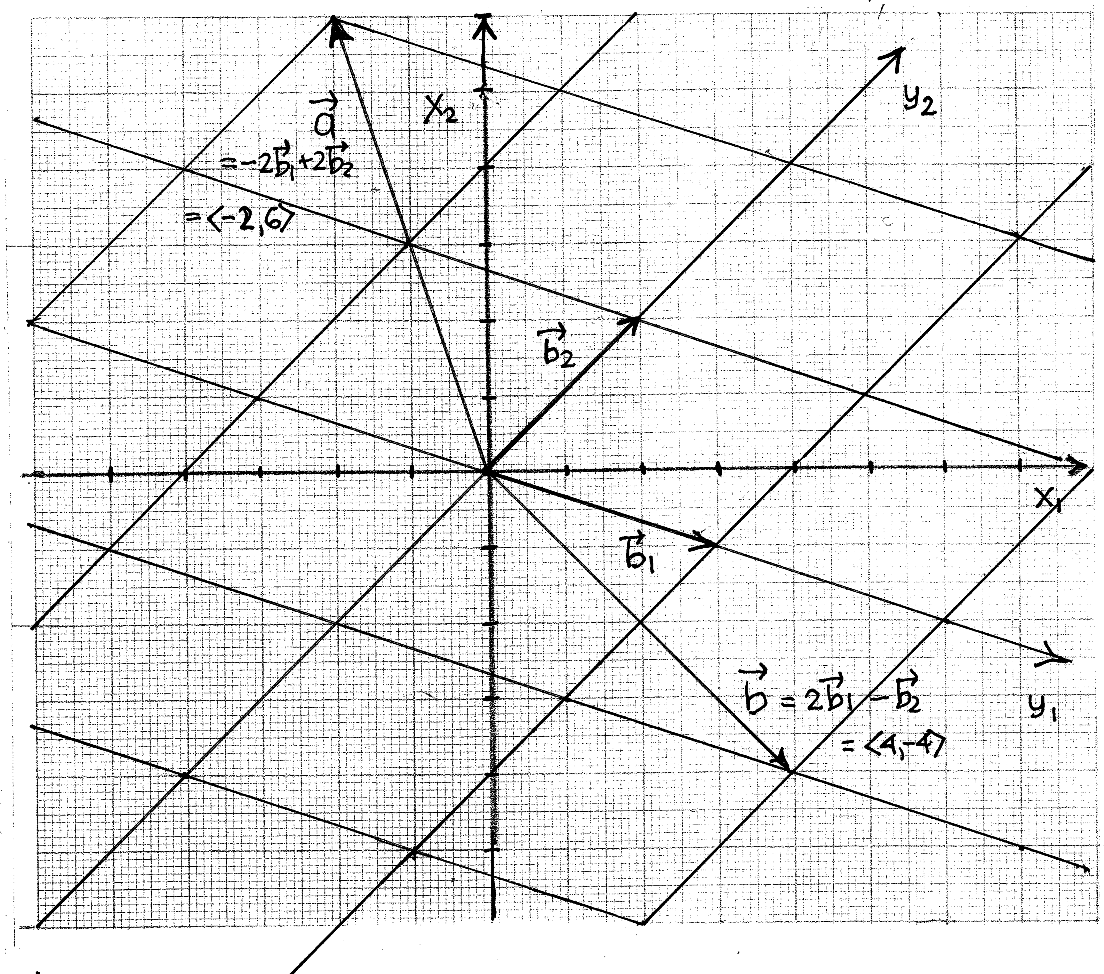
Lastname, Firstname: ______________________



old basis:

$\vec{e}_1 = \langle 1,0 \rangle \quad \vec{e}_2 = \langle 0,1 \rangle$

new basis:

$\vec{b}_1 = \langle 3,-1 \rangle, \vec{b}_2 = \langle 2,2 \rangle$

$$B = \langle \vec{b}_1 | \vec{b}_2 \rangle = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$$

basis changing matrix (columns are new basis vectors)

its inverse:

$$B^{-1} = \begin{bmatrix} \frac{2}{8} & -\frac{2}{8} \\ \frac{1}{8} & \frac{3}{8} \end{bmatrix}$$

linear change of coordinates

$\vec{x}$ in terms of $\vec{y}$: $\vec{x} = B\vec{y}$

$\vec{y}$ in terms of $\vec{x}$: $\vec{y} = B^{-1}\vec{x}$

matrix form:
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{8} & -\frac{2}{8} \\ \frac{1}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

scalar form:

$x_1 = 3y_1 + 2y_2$

$x_2 = -y_1 + 2y_2$

$y_1 = \frac{1}{8}(2x_1 - 2x_2)$

$y_2 = \frac{1}{8}(x_1 + 3x_2)$

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use matrix or scalar form & show work here:

point 1: $\vec{a} = \langle x_1, x_2 \rangle = \langle -2, 6 \rangle \rightarrow \langle y_1, y_2 \rangle = \begin{bmatrix} \frac{1}{8}(2(-2) - 2(6)) \\ \frac{1}{8}((-2) + 3(6)) \end{bmatrix} = \begin{bmatrix} \frac{1}{8}(-4-12) \\ \frac{1}{8}(-2+18) \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \langle -2, 2 \rangle$

point 2: $\langle y_1, y_2 \rangle = \langle 2, -1 \rangle \rightarrow \vec{b} = \langle x_1, x_2 \rangle = \langle 3(2) + 2(-1), -(2) + 2(-1) \rangle$
$= \langle 6-2, -2-2 \rangle = \langle 4, -4 \rangle$

Instructions: Following the example handout, fill in the blanks above. Then reproduce the example graphic with $\vec{e}_1, \vec{e}_2$ then $\vec{b}_1, \vec{b}_2$. Then use a ruler to create the new 4×4 grid of "unit parallelograms" formed with edges $\vec{b}_1, \vec{b}_2$ and their translated vectors as in the example. Draw in the y_1, y_2 axes on this grid and label them. Then graphically locate and draw in the position vectors of point 1 using the x-grid and point 2 using the y-grid. Confirm that the other coordinates you read off from the other grid agree with your computation above. Your graph should have 4 vector arrows representing $\vec{b}_1, \vec{b}_2, \vec{a}, \vec{b}$.