word problem statement:

Just before midday the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70°F. ⟵ $A=70$

At 12 noon the temperature of the body is 80°F ← $t=0$ ← $T(0)=80 \equiv T_0$

and at 1pm it is 75°F. ← $T(1)=75 \equiv T_1$

Assume that the temperature of the body at the time of death ← t_D

was 98.6°F, and ← $T(t_D)=98.6 \equiv T_D$

and that it has cooled in accord with Newton's law. ⟵ $\frac{dT}{dt}=-k(T-A)$

What was the time of death? $t_D=?$

mathematical translation

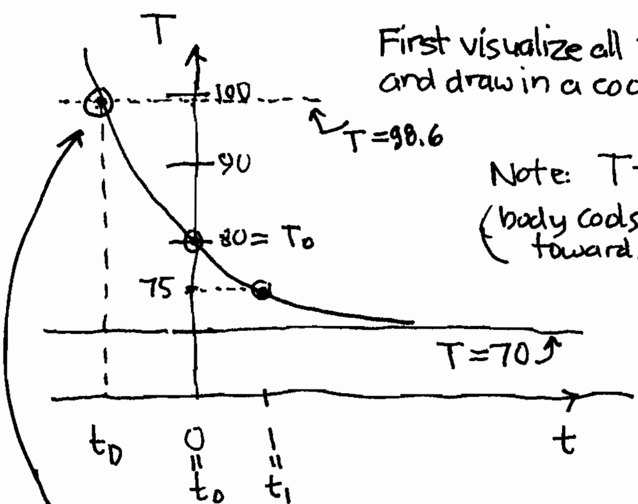
IVP: $\begin{cases} \frac{dT}{dt}=-k(T-A) \\ T(0)=80 \end{cases}$

to determine k: $T(1)=75$

Goal: Find $T(t)$

Then solve condition $T(t_D)=98.6$

Convert t_D to actual clock time.



First visualize all the info and draw in a cooling curve

Note: $T-70>0$ (body cools down towards 70)

this point is very roughly determined by drawing a concave up curve thru the 2 given data points with the given horizontal asymptote

clever:

[If you already know that the temperature difference decays exponentially, notice that $T-70$ decreased by half in 1 hr so the halflife is $T_{1/2}=1$.]

$\left(\frac{Q}{Q_0}=2^{-\frac{t}{T_{1/2}}} \quad \text{compare: } T_{1/2}=1\right)$

$$\frac{dT}{dt}=-k(T-70)$$

$$\int\frac{dT}{T-70}=\int -k\,dt$$

$$\ln(T-70)=-kt+C_1$$

$$e^{\ln(T-70)}=e^{-kt+C_1}$$

$$T-70=\underbrace{e^{C_1}}_{\equiv C}e^{-kt}=Ce^{-kt}$$

$$T=70+Ce^{-kt} \quad \text{(gen soln)}$$

$$T(0)=80=70+C \rightarrow C=10$$

$$T=70+10e^{-kt} \quad \text{(IVP soln)}$$

$$T(1)=75=70+10e^{-k}$$

$$\frac{5}{10}=e^{-k},\ -k=\ln 1/2=\ln(2)^{-1}=-\ln 2$$

$$k=\ln 2$$

$$\boxed{T=70+10e^{-t\ln 2}=70+10\cdot 2^{-t}}$$

$$T(t)=70+10e^{-kt}=98.6,\quad 10e^{-kt}=28.6$$

$$e^{-kt}=2.86,\ -kt=\ln 2.86,\ t=-\frac{\ln 2.86}{k}=-\frac{\ln 2.86}{\ln 2}$$

$$\approx -1.5160 \text{ hr}$$

$$\approx -1 \text{ hr } 30.96 \text{ min}=t_D$$

clock time:

$12{:}00-1{:}31=\boxed{10{:}29\text{ am}}$

"The time of death was approximately 10:29 am"

(Answer the word problem with English sentence!)

Note that we could have solved ALL such cooling problems using 5 parameters: $T_D>T_0>T_1>A$ and t_1, assuming $t_0=0$. We would have obtained a simple formula for t_D expressed in terms of them. Try it!