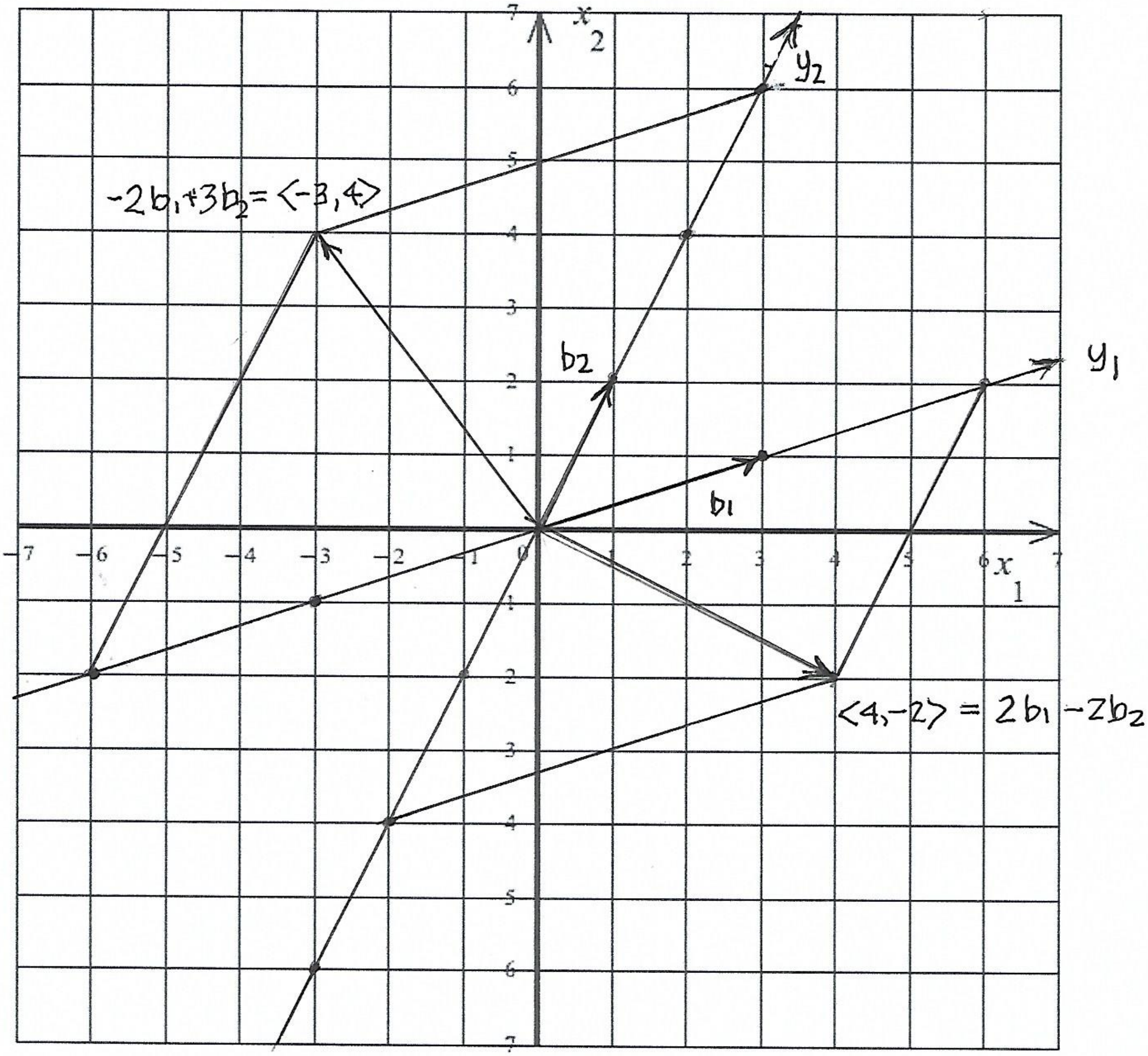
(1)



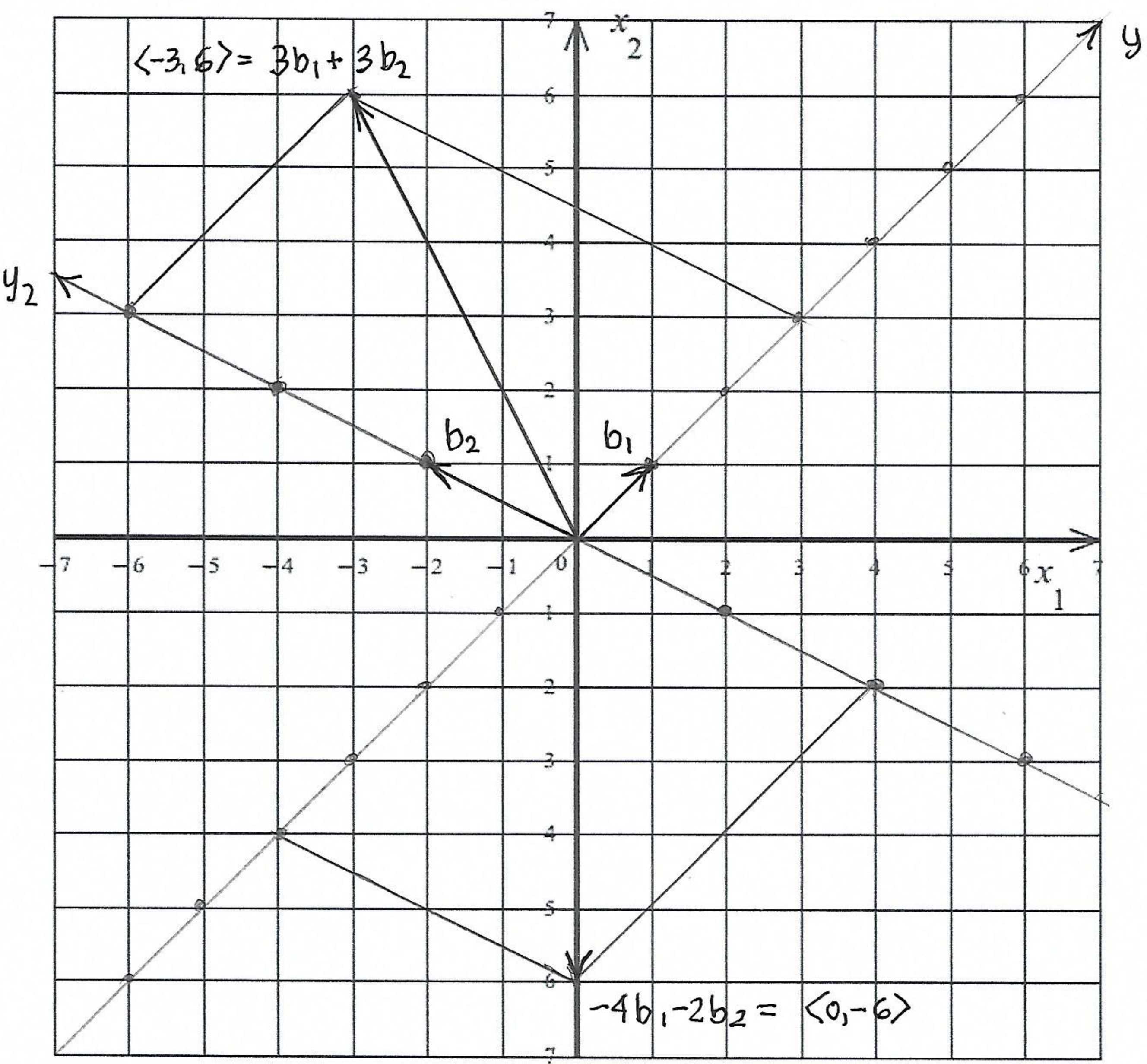
Draw in arrows for the new basis vectors $b_1 = \langle 3, 1\rangle$, $b_2 = \langle 1, 2\rangle$, then extend them in each direction marking off with small bullet circles each multiple tip to tail of these along the new axes to connect with a ruler through these bullet circles to make the new coordinate axes (label them y_1, y_2 at their positive arrowhead ends). Then draw in an arrow for the vector $\langle x_1, x_2\rangle = \langle 4,-2\rangle$ and then draw in lines parallel to each axis from its tip to those axes to form the projection parallelogram, and read off the new coordinates $\langle y_1, y_2\rangle$ *so that* $\langle x_1, x_2\rangle = y_1\, b_1 + y_2\, b_2$. Next reverse the procedure for $\langle y_1, y_2\rangle = \langle -2,3\rangle$ locating the point $\langle x_1, x_2\rangle = y_1\, b_1 + y_1\, b_1$ drawing parallels from the new axes through the tickmarks y_1 and y_2 and draw in its position vector, reading off its old coordinates. Check by matrix multiplication $x = B\,y$, $y = B^{-1}x$ that your graphical readouts are correct.

$B = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$: $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 8+2 \\ -4-6 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ $\quad \langle 4,-2\rangle = 2b_1 - 2b_2$

$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -6+3 \\ -2+6 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $\quad -2b_1 + 3b_2 = \langle -3,4\rangle$

2



Draw in arrows for the new basis vectors $b_1 = \langle 1, 1\rangle$, $b_2 = \langle -2, 1\rangle$, then extend them in each direction marking off with small bullet circles each multiple tip to tail of these along the new axes to connect with a ruler through these bullet circles to make the new coordinate axes (label them y_1, y_2 at their positive arrowhead ends). Then draw in an arrow for the vector $\langle x_1, x_2\rangle = \langle -3, 6\rangle$ and then draw in lines parallel to each axis from its tip to those axes to form the projection parallelogram, and read off the new coordinates $\langle y_1, y_2\rangle$ *so that* $\langle x_1, x_2\rangle = y_1 b_1 + y_2 b_2$. Next reverse the procedure for $\langle y_1, y_2\rangle = \langle -4, -2\rangle$ locating the point $\langle x_1, x_2\rangle = y_1 b_1 + y_2 b_2$ drawing parallels to this point from the new axes through the tickmarks y_1 and y_2 and draw in its position vector, reading off its old coordinates. Check by matrix multiplication $x = B\,y, y = B^{-1}x$ that your graphical readouts are correct.

$$B = \langle b_1 | b_2 \rangle = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \qquad B^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}; \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3+12 \\ 3+6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad \langle -3, 6\rangle = 3b_1 + 3b_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}: \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 \\ -2 \end{bmatrix} = \begin{bmatrix} -4+4 \\ -4-2 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \end{bmatrix} \quad -4b_1 - 2b_2 = \langle 0, -6\rangle$$