

HW Problem BEATING (5.6.1 extra)

$$x'' + x = \cos(1.05t), \quad x(0) = 0, \quad x'(0) = 0$$

$$\text{Note } \omega_0 = 1, \quad \omega = 1.05 = 21/20, \quad \frac{\omega_0 - \omega}{2} = -\frac{1}{40}, \quad \frac{\omega_0 + \omega}{2} = \frac{41}{40}.$$

The soln is proportional to a difference of cosines.

$$\text{Use the identity } \cos A - \cos B = -2 \sin\left(\frac{A-B}{2}\right) \sin\left(\frac{A+B}{2}\right)$$

to re-express the soln as a product of sines.

The beat period is $T_{\text{beat}} = \frac{2\pi}{\left|\frac{\omega_0 - \omega}{2}\right|}$, and the envelope of the soln is

$$\text{given by the 2 curves: } X = \pm \underbrace{\text{Amplitude}} \sin\left(\left|\frac{\omega_0 - \omega}{2}\right|t\right)$$

where "Amplitude" is the coefficient of the product of sines.

Plot the soln and envelope curves for one beat period, showing two beats of the soln oscillation.

Use the Maple template provided on-line to make this easier.