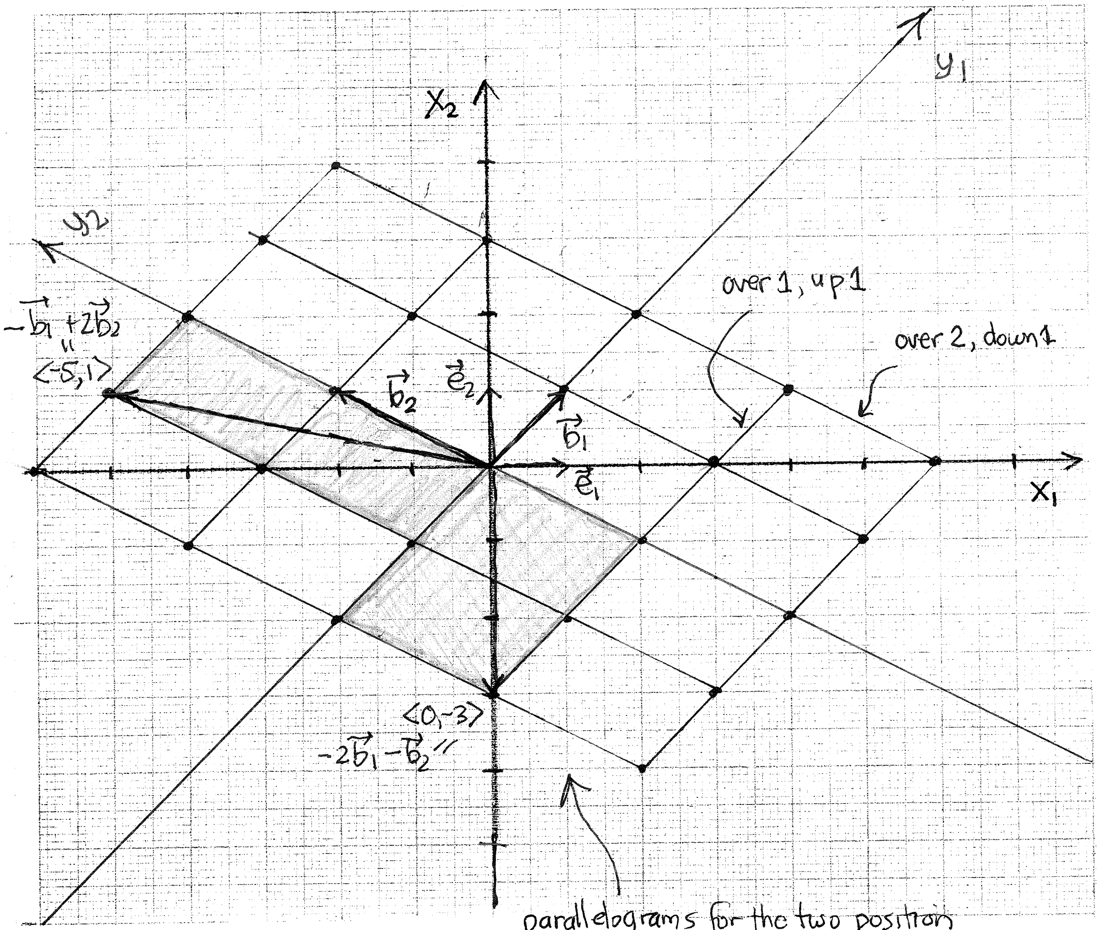
Lastname, Firstname:

key, answer



parallelograms for the two position vectors are shaded in to make obvious the multiples of $\vec{b}_1$ and $\vec{b}_2$ along edges from origin

old basis:

$\vec{e}_1=\langle 1,0\rangle \quad \vec{e}_2=\langle 0,1\rangle$

new basis:

$\vec{b}_1=\langle 1,1\rangle, \vec{b}_2=\langle -2,1\rangle$

$B = \langle \vec{b}_1 | \vec{b}_2 \rangle$

$= \begin{bmatrix} \begin{pmatrix}1\\1\end{pmatrix} & \begin{pmatrix}-2\\1\end{pmatrix} \end{bmatrix}$ columns!

basis changing matrix (columns are new basis vectors)

its inverse:

$$B^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

linear change of coordinates

$\vec{x}$ in terms of $\vec{y}$: $\vec{x} = B\vec{y}$

$\vec{y}$ in terms of $\vec{x}$: $\vec{y} = B^{-1}\vec{x}$

matrix form: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

scalar form: $\left.\begin{aligned} x_1 &= y_1 - 2y_2 \\ x_2 &= y_1 + y_2 \end{aligned}\right\}$ $\qquad$ $\left\{\begin{aligned} y_1 &= \tfrac{1}{3}(x_1+2x_2) \\ y_2 &= \tfrac{1}{3}(-x_1+x_2) \end{aligned}\right.$

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point 1: $\langle x_1, x_2\rangle = \langle -5, 1\rangle \rightarrow \langle y_1, y_2\rangle = \langle \tfrac{1}{3}(-5+2(1)), \tfrac{1}{3}(+5+1)\rangle = \langle -1, 2\rangle$

point 2: $\langle y_1, y_2\rangle = \langle -2, -1\rangle \rightarrow \langle x_1, x_2\rangle = \langle -2-2(-1), -2+-1\rangle = \langle 0, -3\rangle$

Instructions: Following the example handout, fill in the blanks above. Then reproduce the example graphic with $\vec{e}_1, \vec{e}_2$ then $\vec{b}_1, \vec{b}_2$. Then use a ruler to create the new 4×4 grid of "unit parallelograms" formed with edges $\vec{b}_1, \vec{b}_2$ and their translated vectors as in the example. Draw in the y_1, y_2 axes on this grid and label them. Then graphically locate and draw in the position vectors of point 1 using the x-grid and point 2 using the y-grid. Confirm that the other coordinates you read off from the other grid agree with your computation above.