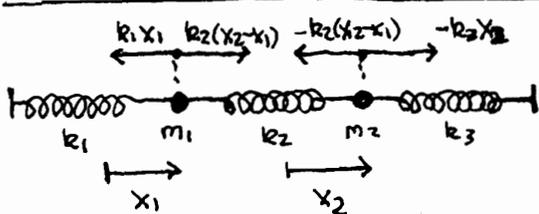


ERP3: 7.4 3,9 2 mass - 3 spring system exercise



$$\begin{bmatrix} m_1 x_1'' \\ m_2 x_2'' \end{bmatrix} = \begin{bmatrix} -(k_1+k_2) & k_2 \\ k_2 & -(k_2+k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B_0 \cos \omega t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$m_1 = 1, m_2 = 2$
 $k_1 = 1, k_2 = 2 = k_3$
 $B_0 = 120, \omega = 3$

- a) Plug in values, put into standard form: $\vec{x}'' = A\vec{x} + \vec{f}$
- b) Find eigenvalues λ_1, λ_2 of A : $|\lambda_1| < |\lambda_2|$ and eigenvectors $B = \langle \vec{b}_1 | \vec{b}_2 \rangle$ and $A_B = B^{-1}AB$.
- c) Transform to new variables $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ by $\vec{x} = B\vec{y}$: $\vec{y}'' = A_B\vec{y} + B^{-1}\vec{f}$
- d) Solve decoupled DEs for y_1, y_2 : $\vec{y} = \vec{y}_h + \vec{y}_p$.
- e) Solve initial conditions for $\vec{x} = B\vec{y}$.
- f) Express the result as $\vec{x} = y_{1h}\vec{b}_1 + y_{2h}\vec{b}_2 + B\vec{y}_p$.

Identify $\left\{ \begin{array}{l} \text{frequencies: } \omega_1 \quad \omega_2 \quad \omega_3 = 3 \\ \text{periods: } T_1 \quad T_2 \quad T_3 \\ \text{amplitude ratios:} \end{array} \right.$ (common period of 2π)

includes sign $\rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

- g) Return to \vec{y}_p calculation and redo for a general frequency ω not equal to ω_1, ω_2 by letting $3 \rightarrow \omega, 9 \rightarrow \omega^2$ in those calculations.

Recombine: $\vec{x}_p = B\vec{y}_p = \begin{bmatrix} a(\omega) \\ b(\omega) \end{bmatrix} \cos \omega t$

Plot $|a(\omega)|, |b(\omega)|$ amplitudes of response functions.

- h) Add damping term: $m_i x_i'' = \dots - c_i x_i'$ where for simplicity $c_i = m_i$.

Calculate new response functions:

$$\vec{x}_p = B\vec{y}_p = \begin{bmatrix} A_1(\omega) \cos(\omega t - \delta_1) \\ A_2(\omega) \cos(\omega t - \delta_2) \end{bmatrix}$$

Plot $A_1(\omega), A_2(\omega)$ (response amplitudes).

- i) Redo using reduction of order for $\langle x_1, x_2, x_3, x_4 \rangle = \langle x_1, x_2, x_1', x_2' \rangle$. Find 4×4 matrix coefficient eigenvalues and eigenvectors for zero damping and this nonzero damping case. Compare.