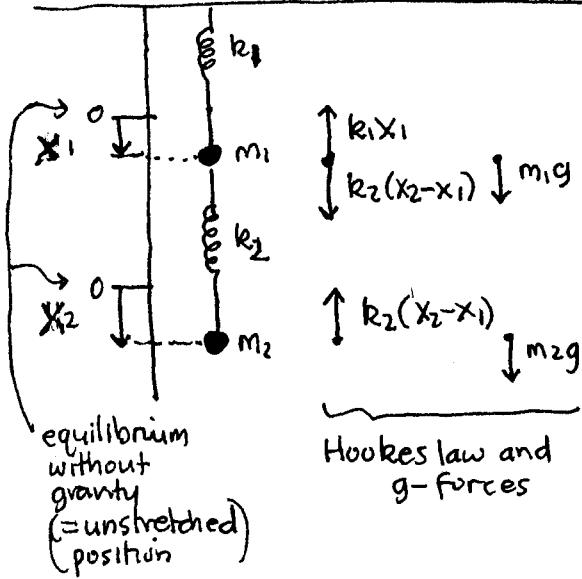


## 2 spring-2 mass system with gravity: exploration



$$m_1 \ddot{x}_1'' = -k_1 x_1 + k_2(x_2 - x_1) + m_1 g = -(k_1 + k_2)x_1 + k_2 x_2 + m_1 g$$

$$m_2 \ddot{x}_2'' = -k_2(x_2 - x_1) + m_2 g = k_2 x_1 - k_2 x_2 + m_2 g$$

$$\begin{bmatrix} m_1 \ddot{x}_1'' \\ m_2 \ddot{x}_2'' \end{bmatrix} = \begin{bmatrix} -(k_1 + k_2) & k_2 \\ k_2 & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} m_1 g \\ m_2 g \end{bmatrix}$$

To make simple enough numbers in the computations take  $m_1 g = 1 \text{ lb}$ ,  $m_2 g = 1 \text{ lb}$

$$k_1 = 3 \text{ lbs}/\text{ft}, k_2 = 2 \text{ lbs}/\text{ft}$$

and reduce Earth gravity from  $g = 32 \text{ ft/sec}^2$  to  $g = 1 \text{ ft/sec}^2$  [This is equivalent to using  $T = \sqrt{32} t$  as a new time variable in actual Earth gravity, slowing things down but not changing the motion otherwise.]

- Evaluate the matrix eqs of motion by plugging in all the numbers.
- Set the time derivatives to zero and solve for  $x_1, x_2$  to find the new stretched equilibrium positions taking gravity into account. [These are integers when expressed in inches!]
- If you pull on the second mass with a force of 2lbs, the springs will stretch to new positions which can be found by increasing  $m_2 g$  from 1lb to 3lbs. Resolve for  $x_1$  and  $x_2$  and use these as initial conditions  $\underline{x}(0)$  and  $\dot{\underline{x}}(0)$  at rest when they are released:  $x_1'(0) = 0 \approx x_2'(0)$ . [also integers when expressed in inches!]
- The eqs of motion are:  $\underline{x}'' = \underline{A} \underline{x} + \underline{f}$ . Find the eigenvalues of  $\underline{A}$  and then the corresponding eigenvectors, rescaled to have integer entries. List them in increasing order of their eigenvalue absolute values and form the basis changing matrix:  $\underline{B} = \text{augment } (\underline{E}_1, \underline{E}_2)$  so that  $\underline{A}\underline{B} = \underline{B}^{-1} \underline{A} \underline{B} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ .
- The change of variables  $\underline{x} = \underline{B} \underline{y}$ ,  $\underline{y} = \underline{B}^{-1} \underline{x}$  leads to:  $\underline{y}'' = \underline{A} \underline{B} \underline{y} + \underline{B}^{-1} \underline{f}$ . Evaluate  $\underline{B}^{-1}$  and  $\underline{B}^{-1} \underline{f}$  and then write out the decoupled eqns for  $y_1$  and  $y_2$ . Solve them with the above initial conditions. Express your solution in both forms:  $\underline{x} = \underline{B} \underline{y}$  (multiplied out)  $= y_1 \underline{E}_1 + y_2 \underline{E}_2$  (left as a linear combination)

Notice that the amplitude of the oscillating term in  $y_2$  is 12 times smaller than the oscillating term in  $y_1$  — this is because the initial conditions for  $x_1(0)$  and  $x_2(0)$  are almost in the same ratio as the entries of  $\underline{E}_1$ , so only a small contribution to the soln is made by the second mode associated with  $\underline{E}_2$ , i.e., primarily the  $\underline{E}_1$  mode is excited.

- Your soln is of the form:  $x_1 = \underline{x}_1 + a_1 \cos \omega_1 t + a_2 \cos \omega_2 t$   $x_2 = \underline{x}_2 + a_3 \cos \omega_1 t + a_4 \cos \omega_2 t$  (check your soln with dsolve) on deas, init

plot([ $x_1, x_2, \underline{x}_1 + a_1 \cos \omega_1 t, \underline{x}_2 + a_3 \cos \omega_1 t, a_2 \cos \omega_2 t, a_4 \cos \omega_2 t, \underline{x}_1 + |a_1| + |a_2|, \underline{x}_1 + |a_1| - |a_2|, \underline{x}_2 + |a_3| + |a_4|, \underline{x}_2 + |a_3| - |a_4|$ ], color = red, blue, black, black, red, blue,  $\underline{x}_2 + |a_3| + |a_4|, \underline{x}_2 + |a_3| - |a_4|$ ], all grey)

for  $t = 0..6 \times 2\pi t$

The black curves are the first mode, but the small oscillations randomly make the red-blue soln curves wander between the grey lines.