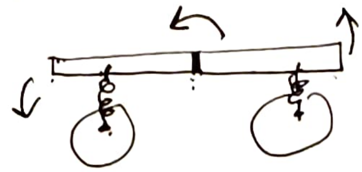
Coupled mass spring systems: 2-axle car model

A two axle car shock absorber system couples the car mass to the two axle springs (shock absorbers). Unequal stretching/compressing of front and back leads to rotation of the angle of the body from the horizontal.



We divide the car front and back axle height displacements into the overall displacement of the car center of mass and the rotation of the body about it.

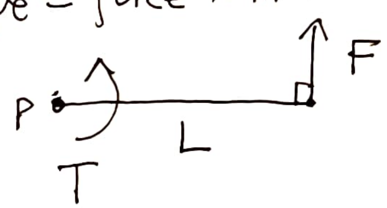
It is relatively easy to understand the model if you accept the basic equations of motion for linear and angular motion.

Basic physical quantities

linear motion: $F = mx''$ force = mass × linear acceleration

angular motion: $T = I\theta''$ torque = moment of inertia × angular acceleration

torque = force × "moment arm"

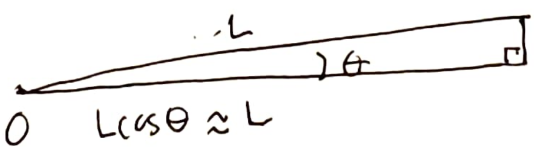


torque about point P equals moment arm L times the perpendicular force F applied at the end of the moment arm

$T = LF$

Small angle approximation (to obtain linear DEs)

$\theta << 1$:



$L\sin\theta \approx L\theta$

$L\cos\theta \approx L$

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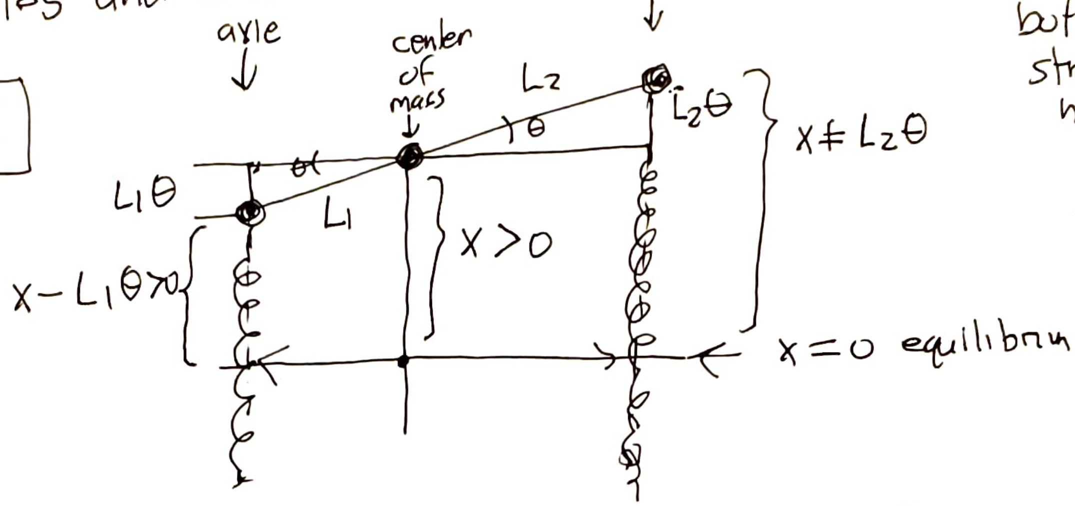
2 axles and center of mass:

FORCE



both springs stretched here: $x>0$

so pull <u>down</u>

$$F_1 = -k_1(x-L_1\theta) < 0$$

restoring force down

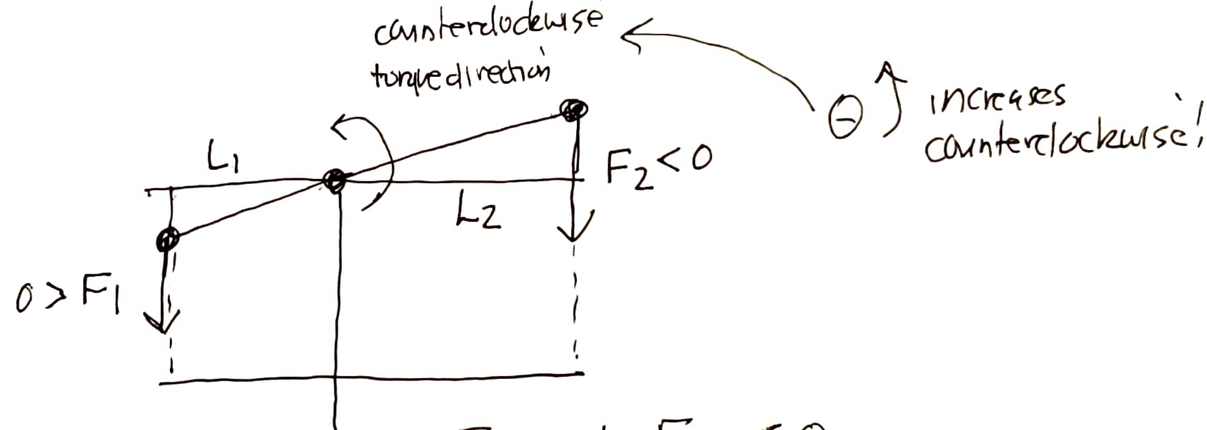
$$F_2 = -k_2(x+L_2\theta) < 0$$

restoring force down

net upward force (x axis up!):

$$F = F_1 + F_2 = -k_1(x-L_1\theta) - k_2(x+L_2\theta)$$
$$= -(k_1+k_2)x + (k_1L_1 - k_2L_2)\theta = mx''$$

TORQUE



$$T_1 = L_1(-F_1) > 0$$

positive contribution

$$T_2 = L_2F_2 < 0$$

negative contribution

net counterclockwise torque:

$$T = T_1 + T_2 = -L_1F_1 + L_2F_2$$
$$= L_1(k_1)(x-L_1\theta) + L_2(-k_2)(x+L_2\theta)$$
$$= (L_1k_1 - L_2k_2)x - (k_1L_1^2 + k_2L_2^2)\theta = I\theta''$$

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eqns of motion:

$$m x'' = -(k_1+k_2)x + (k_1L_1 - k_2L_2)\theta$$
$$I\theta'' = (k_1L_1 - k_2L_2)x - (k_1L_1^2 + k_2L_2^2)\theta$$

standard matrix form:

$$\begin{bmatrix} x'' \\ \theta'' \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{(k_1+k_2)}{m} & \frac{k_1L_1-k_2L_2}{m} \\ \frac{k_1L_1-k_2L_2}{I} & -\frac{(k_1L_1^2+k_2L_2^2)}{I} \end{bmatrix}}_{A} \begin{bmatrix} x \\ \theta \end{bmatrix}$$

$$\begin{matrix} \lambda_2 < \lambda_1 < 0 \\ \| \quad\quad \| \\ -\omega_2^2 \quad -\omega_1^2 \end{matrix}$$

2 negative eigenvalues

order abs values!

$|\lambda_1| < |\lambda_2|$

slow fast

frequencies ordered: $\omega_1 = \sqrt{-\lambda_1} < \omega_2 = \sqrt{-\lambda_2}$

$b_1 = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$ $\frac{b_{11}}{b_{21}} > 0$ same sign (tandem)

$b_2 = \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix}$ $\frac{b_{12}}{b_{22}} < 0$ opp signs (accordian)

very similar to 2 mass – 3 spring system properties.

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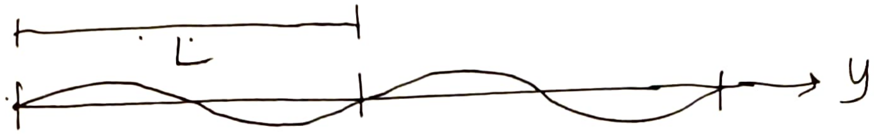
Sinusoidal height pavement delivers sinusoidal force to this system

Let y be horizontal position of car



height $z = a \cos\left(2\pi \frac{y}{L}\right)$ profile of pavement.
(when $y = L$, cosine returns to 1)

but if $y = vt$, v = speed car

then $z = a \cos\left(\underbrace{\frac{2\pi v}{L}}\, t\right)$

$\omega = \frac{2\pi v}{L}$ or $v = \frac{L}{2\pi}\omega$

If we choose the speed so that $\omega = \omega_1$ or $\omega = \omega_2$

then $\underbrace{v_1 = \frac{L}{2\pi}\omega_1, \quad v_2 = \frac{L}{2\pi}\omega_2}_{\text{units ft/sec}}$

$\frac{1\,ft}{sec} = \left(\frac{1\,ft}{sec}\right)\left(\frac{60\,sec}{1\,min}\right)\left(\frac{60\,min}{1\,hr}\right)\left(\frac{1\,mi}{5280\,ft}\right) = \underbrace{\frac{15}{22}\frac{mi}{hr} \approx 0.68\text{ mph}}_{\text{conversion factor for interpretation}}$

frequency units ω = radians/sec

$\downarrow$

$\frac{\omega}{2\pi}$ = "cycles"/sec $\equiv$ Hertz (Hz)

for interpretation!!

(no one understands radian numbers!!)

clearly we want decimal answers here! evaluate all numbers to decimal form but in maple do two steps: evalf(...) $\to$ 10 digits $\longrightarrow$ evalf(..., 5) round off.