

3.1 linear systems of equations: elimination

(1)

high school elimination example:

$$\begin{cases} 2x + 7y + 3z = 11 \\ x + 3y + 2z = 2 \\ 3x + 7y + 9z = -12 \end{cases}$$

switch to make elimination arithmetic easier (convenient)

$$\begin{array}{l} \left\{ \begin{array}{l} x + 3y + 2z = 2 \\ 2x + 7y + 3z = 11 \\ 3x + 7y + 9z = -12 \end{array} \right. \quad \rightarrow \quad \left. \begin{array}{l} 2x + 7y + 3z = 11 \\ -2x - 6y - 4z = -4 \\ \hline y - z = 7 \end{array} \right. \quad \text{eliminate } x \text{ first from last 2 eqns} \\ \qquad \qquad \qquad \left. \begin{array}{l} 3x + 7y + 9z = -12 \\ -3x - 9y - 6z = -6 \\ \hline -2y + 3z = -18 \end{array} \right. \\ \left. \begin{array}{l} x + 3y + 2z = 2 \\ y - z = 7 \\ -2y + 3z = -18 \end{array} \right. \quad \xrightarrow{*2} \quad \left. \begin{array}{l} 2y - 2z = 14 \\ -2y + 3z = -18 \\ \hline z = -4 \end{array} \right. \quad \text{eliminate } y \text{ next from last eqn} \\ \left. \begin{array}{l} x + 3y + 2z = 2 \\ y - z = 7 \\ z = -4 \end{array} \right. \quad \xrightarrow{x(3)} \quad \left. \begin{array}{l} x = 2 - 3(3) - 2(-4) \\ = 2 - 9 + 8 = -1 \\ y = 7 + (-4) = 3 \\ \downarrow \\ x = 1, y = 3, z = -4 \end{array} \right. \quad \text{next backsub from bottom up} \end{array}$$

successive eliminations now complete

(notice "triangular" shape of nonzero terms on LHS)

$x = 1, y = 3, z = -4$

unique soln

Interpretation: 3 planes intersect in a single point

These steps will be programmed into a matrix reduction algorithm to solve any linear system of equations

Then we can forget this technique.

"inconsistent" system: 3 planes with no common intersection

see MAPLE

3.1 linear systems of equations: elimination

(2)

Linear DEs are linear in the unknown and its derivatives. Imposing initial conditions leads to linear systems of equations for the arbitrary constants since the general solutions are linear in those constants.

example

$$\text{IVP: } y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

$$\text{gen soln: } y = C_1 e^{-x} + C_2 e^{-2x}$$

$$y' = -C_1 e^{-x} - 2C_2 e^{-2x}$$

impose initial conditions

$$\begin{aligned} y(0) &= C_1 + C_2 = 1 \\ y'(0) &= -C_1 - 2C_2 = 1 \end{aligned}$$

solve:

$$C_1 = 3, \quad C_2 = -2$$

never a final result for
any calculation:
ALWAYS BACKSUB!

$$y = 3e^{-x} - 2e^{-2x}$$

This is "the solution" of
the IVP

See Maple

systems of linear DEs need even more linear mathematics
so we have to take a serious detour into enough
"linear algebra" to return to systems

The next example is only a PREVIEW
of where we are headed
to motivate why linear algebra is needed.