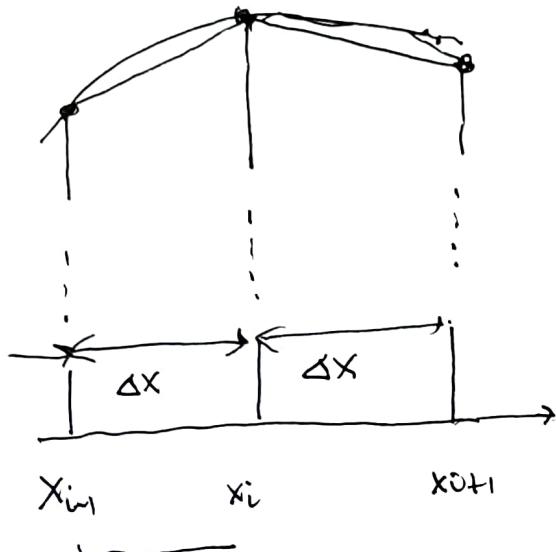


2.A) numerical DE solving: Euler's method

①

In Calc 2 we were exposed to some numerical methods to approximate definite integration which is a special kind of DE solving

$$y = f(x)$$

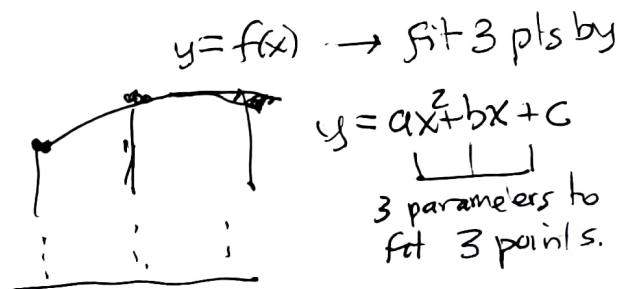


$$\Delta A \approx \frac{1}{2}(f(x_i) + f(x_{i-1}))\Delta x$$

trapezoidal area
from linear approx to graph

The trapezoidal rule approximates the increment of area by "fitting" a linear curve to the graph and using the area under it for the approximation.

This is much improved by Simpson's rule which instead fits a quadratic curve to the graph



$y = f(x) \rightarrow$ fit 3 pts by

$$y = ax^2 + bx + c$$

3 parameters to fit 3 points.

use its area to approximate area increment over pairs of subintervals.

Euler's (forward) Method

is analogous to the trapezoidal rule and can be improved by many more sophisticated methods like Simpson's rule does for Euler. Several textbook sections discuss this.

We just want to understand the simplest approach as a window into numerical solution of DEs.

Approximate solution to IVP:

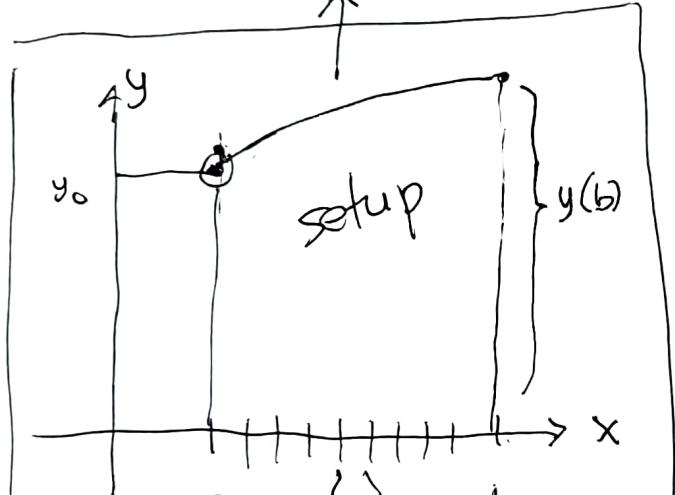
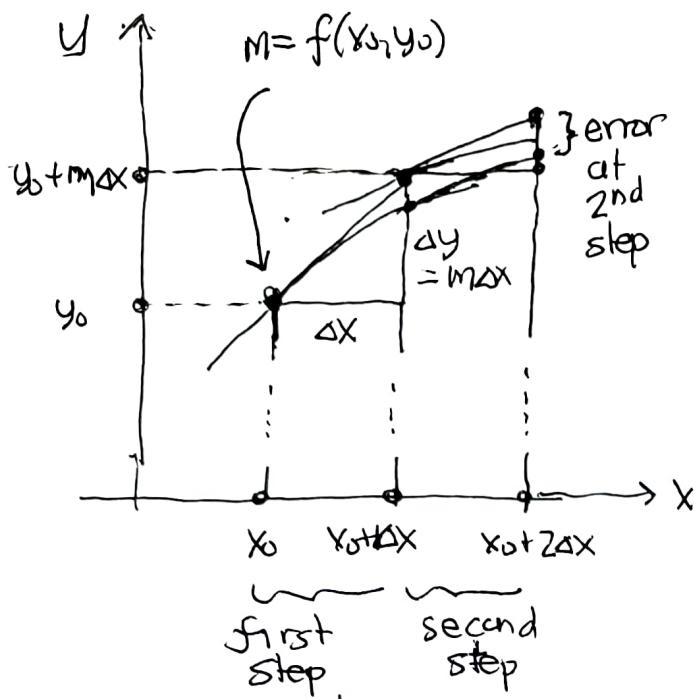
$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0 \text{ on interval } x_0 \leq x \leq x_f$$

at n discrete points along the way

Exercises. Evaluate $y(x_f)$ approximately.

2.4 numerical DE solving: Euler's method

(2)



Same setup as for approximate integration.

Goal. Approximate: $y(b)$

We start at an initial data point and simply follow the tangent line to a solution curve one step, arriving at the next point on our approximate solution curve.

Repeat in a loop for successive steps. Let $h \equiv \Delta x$

$$(x_0, y_0) \rightarrow (x_0 + h, y_0 + f(x_0, y_0)h) \\ \equiv (x_1, y_1)$$

$$(x_1, y_1) \rightarrow (x_2, y_2) = \dots$$

$$(x_i, y_i) \rightarrow (x_{i+1}, y_{i+1}) \\ \equiv (x_i + h, y_i + h f(x_i, y_i))$$

Repeat this step till you get to (x_n, y_n) .

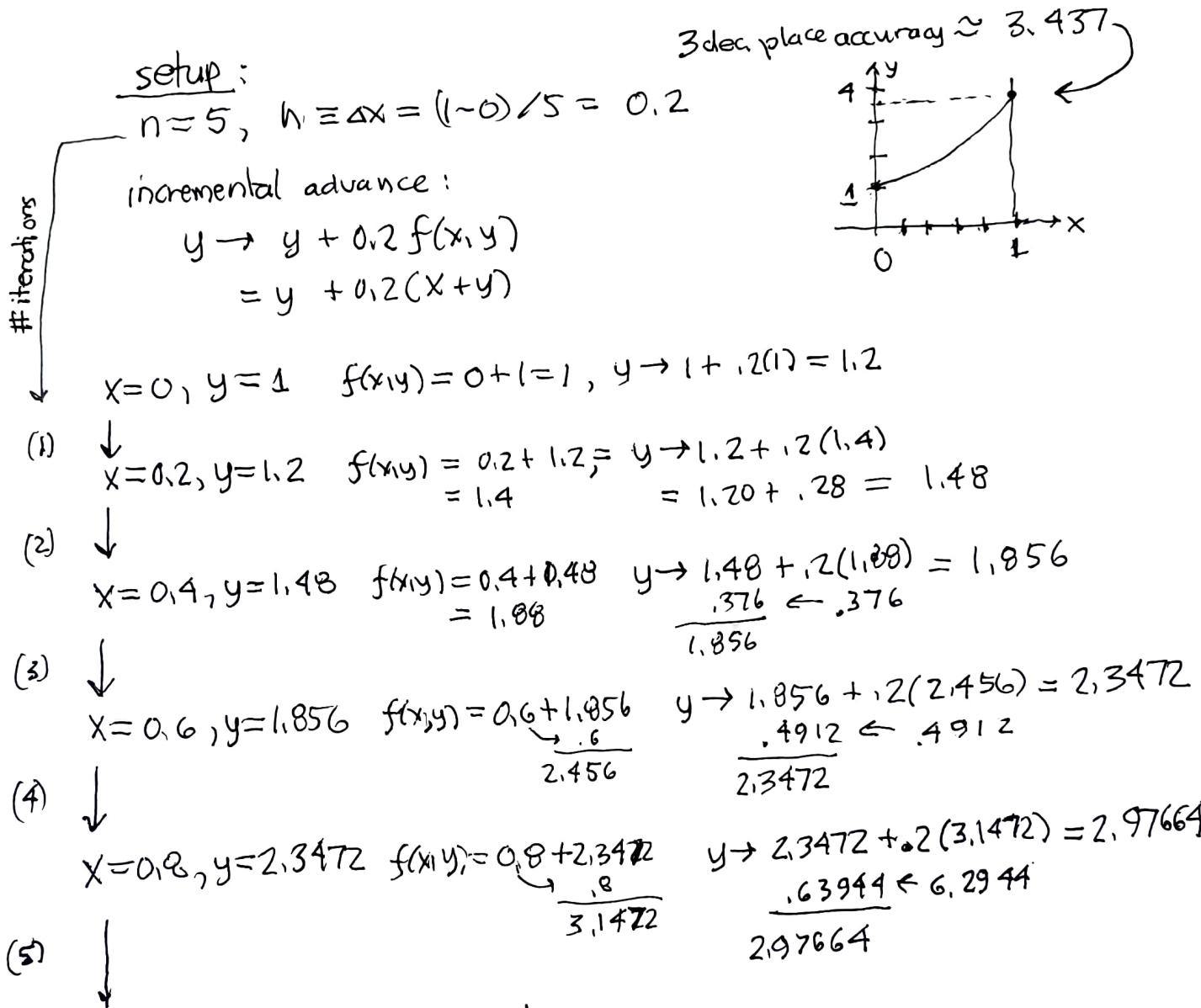
y_n is approximate solution at $x = x_n$.

2.4

numerical DE solving: Euler's method

3

Example $\frac{dy}{dx} = x+y \equiv f(x,y)$, $y(0)=1$ on interval $0 \leq x \leq 1$.
 easily solved: $y = -(x+1) + 2e^x \rightarrow y(1) = -2+2e^1 \approx 3.4365$



$$x=1.0, y=3.61608$$

$\ll 3.4365$

much lower, concave up curve
pulls up from straight line
approximations



ridiculous to do this by hand!

computers invented to do this! (long before Facebook etc)

To improve just increase # n of iterations (or better method!)

see Maple