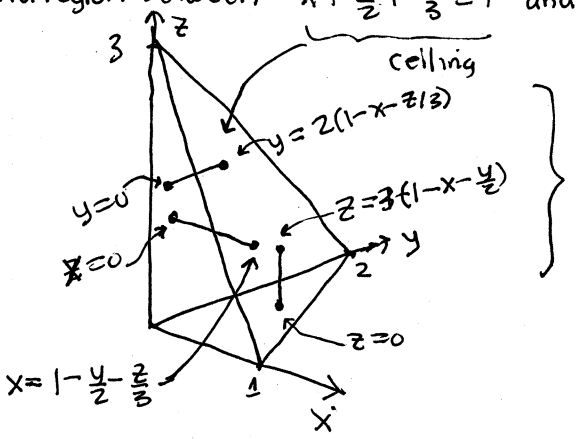
Examples of iterating triple integrals

Solid region between $x+\frac{y}{2}+\frac{z}{3}=1$ and $x=0$, $y=0$, $z=0$ planes. Find Volume.

ceiling; side walls; floor

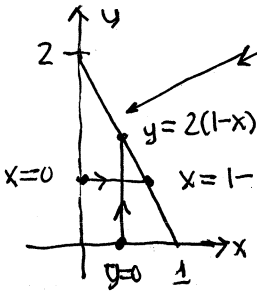


innermost integral limits come from starting and stopping values of variable along cross-section line segments.

outer double integral limits come from projections of solid regions onto 3 coordinate planes, only need 2d diagrams

z first: $z=0..3(1-x-\frac{y}{2})$

$x+\frac{y}{2}+\frac{z}{3}=1$ intersects $z=0$ at $x+\frac{y}{2}=1$



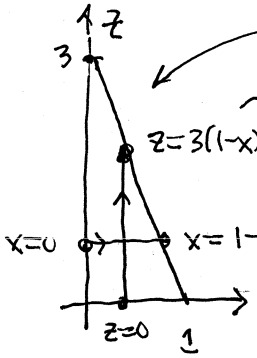
solved for each variable

$$\int_0^1\int_0^{2(1-x)}\int_0^{3(1-x-\frac{y}{2})} 1\ dz\,dy\,dx$$

$$\int_0^2\int_0^{1-\frac{y}{2}}\int_0^{3(1-x-\frac{y}{2})} 1\ dz\,dx\,dy$$

y first: $y=0..2(1-x-\frac{z}{3})$

$x+\frac{y}{2}+\frac{z}{3}=1$ intersects $y=0$ at $x+\frac{z}{3}=1$



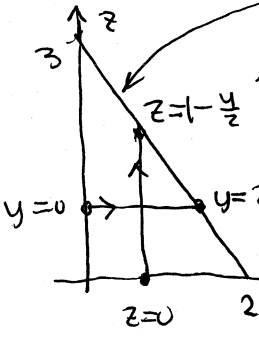
solved for each variable

$$\int_0^1\int_0^{3(1-x)}\int_0^{2(1-x-z/3)} 1\ dy\,dz\,dx$$

$$\int_0^3\int_0^{1-z/3}\int_0^{2(1-x-z/3)} 1\ dy\,dx\,dz$$

x first: $x=0..1-y/2-z/3$

$x+\frac{y}{2}+\frac{z}{3}=1$ intersects $x=0$ at $\frac{y}{2}+\frac{z}{3}=1$



solved for each variable

$$\int_0^3\int_0^{2(1-z/3)}\int_0^{1-y/2-z/3} 1\ dx\,dy\,dz$$

$$\int_0^2\int_0^{1-y/2}\int_0^{1-y/2-z/3} 1\ dx\,dz\,dy$$

all six integrals give same result.