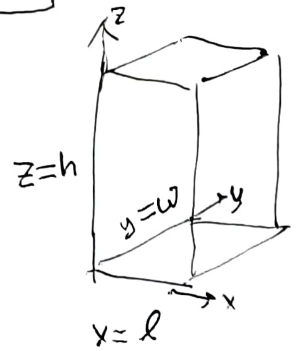
$\boxed{14.R.64}$ Maximize the volume of a box with fixed girth.



Girth: $G = 2(x+y) + z = 2(\ell + w) + h$

Volume: $V = xyz = \ell w h$ (maximize)

[note interchange symmetry of (x,y) so solution must have $y = x$]

choose variable names as you like

Girth = lateral circumference plus height

Eliminate z: $z = G - 2(x+y)$

Volume: $V = xy(G - 2(x+y)) = Gxy - 2(x^2y + xy^2)$

Critical points:

$$V_x = Gy - 2(2xy + y^2) = y(G - 2(2x+y)) = 0$$
$$V_y = Gx - 2(x^2 + 2xy) = x(G - 2(x+2y)) = 0$$

$$2(2x+y) = G$$
$$2(x+2y) = G$$
$$-:\quad 2(x-y) = 0 \rightarrow y = x$$

$$2(x + 2x) = G$$
$$6x = G$$
$$x = G/6 = y$$

$$z = G - 2\left(\frac{G}{6} + \frac{G}{6}\right) = G\left(1 - \frac{2}{3}\right) = \frac{G}{3}$$
$$= 2x = 2y$$

$$V = \left(\frac{G}{6}\right)\left(\frac{G}{6}\right)\left(\frac{G}{3}\right) = \frac{G^3}{108} = 2x^3$$

The height is twice the equal length and width which in turn are a sixth of the girth, while the volume is the girth cubed over 108 or twice the ~~cube~~ of ~~the~~ equal length and width.

For the USPS set $G = 108$ inches, so $x = y = 18$ inches, $z = 36$ inches