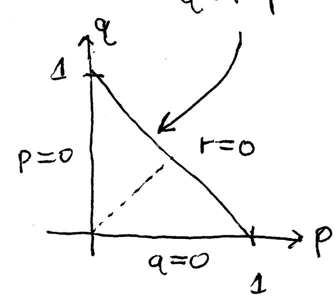
max-min with boundary

maximize: $P = 2pq + 2pr + 2rq$, $p \geq 0, q \geq 0, r \geq 0$

subject to constraint $p+q+r=1$ $\longrightarrow$ eliminate r! $r = 1-p-q \geq 0$

$q = 1-p$

$P = 2pq + 2(p+q)(1-p-q)$ on interior and border of triangular region



interior critical points

$P_p = 2q + 2(1)(1-p-q) + 2(p+q)(-1)$

$= \cancel{2q} + 2 - 2p - \cancel{2q} - 2p - 2q$ $= 0$

$= 2(1-2p-q)$ switch p,q

$P_q = \cdots = 2(1-p-2q)$ $= 0$

critical point:

$2p + q = 1 \rightarrow q = 1-2p$

$p + 2q = 1$

$\hookrightarrow p + 2(1-2p) = 1$

$p + 2 - 4p$

$2 - 3p$

$2 - 1 = 3p \rightarrow p = 1/3$

$q = 1 - 2(1/3) = 1/3$

$r = 1 - (\frac{1}{3}) - (\frac{1}{3}) = 1/3$

$P_{pp} = 2(-2) = -4 < 0$

$P_{qq} = 2(-2) = -4 < 0$

$P_{pq} = -2$

$P_{pp}P_{qq} - P_{pq}^2 = (-4)^2 - (-2)^2 > 0$ local max.

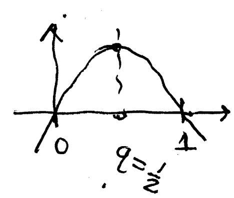
$(p, q) = (1/3, 1/3)$ only critical pt.

$P = 2(\frac{1}{3})(\frac{1}{3}) + 2(\frac{1}{3}+\frac{1}{3})(\frac{1}{3}) = \frac{2+4}{9} = \frac{6}{9} = \boxed{2/3}$

Boundary max-problems

$p = 0$: $P = 0 + 2(0+q)(1-0-q) = 2q(1-q)$

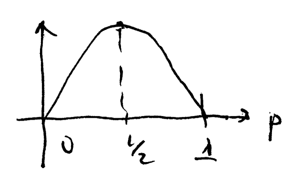
$0 \leq q \leq 1$



$P|_{q=1/2} = 2(\frac{1}{2})(1-\frac{1}{2}) = \boxed{1/2}$ ← local max

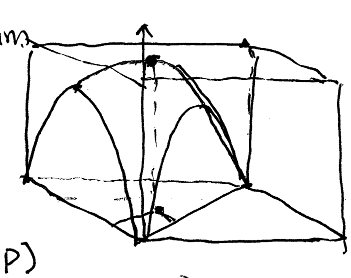
$q = 0$: by symmetry, identical calculation

$r = 0$: $P = 2p\underbrace{(1-p)}_{q} + 0 = 2p(1-p)$: same calculation



$P|_{p=1/2} = 2(\frac{1}{2})(1-\frac{1}{2}) = \boxed{1/2}$ ← local max

$\boxed{P = 2/3 \text{ at } (p,q,r) = (1/3, 1/3, 1/3)}$ is maximum



3d plot makes it obvious

Maple: > plot3d(2pq + 2(p+q)(1-p-q),
p=0..1, q=0..(1-p))