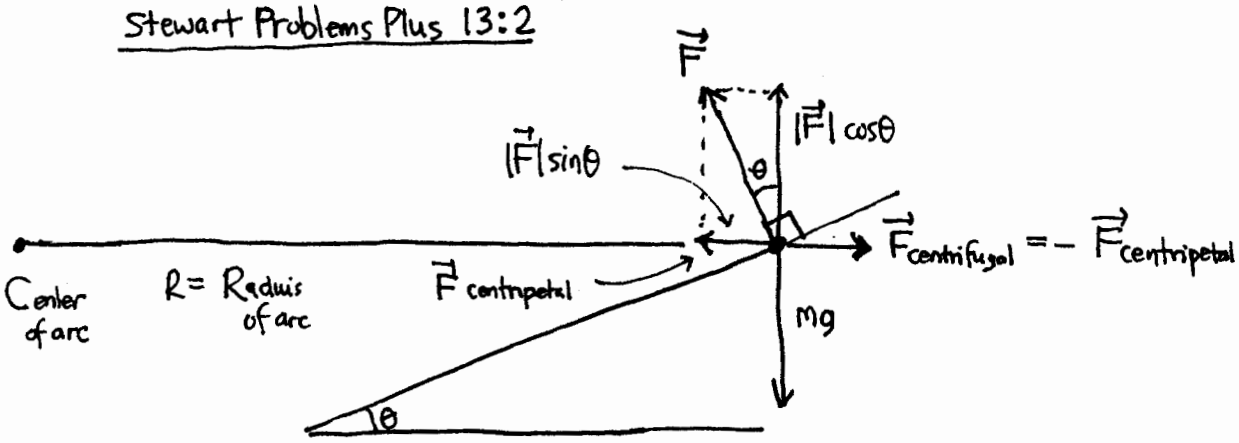
Stewart Problems Plus 13:2



g-force balance (vertical): $|\vec{F}|\cos\theta = mg \rightarrow |\vec{F}| = \frac{mg}{\cos\theta}$ (force exerted by road on vehicle.)

centripetal force acceleration equation (horizontal):

$$|\vec{F}|\sin\theta = ma_\perp = m\frac{v_R^2}{R}$$

v_R = speed of car when no slippage occurs.

(This is equivalent to balancing the centrifugal force (outward) with the inward component of the road force.)

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v_R^2}{R}$$

$$g\tan\theta = \frac{v_R^2}{R}$$

$$\boxed{v_R^2 = Rg\tan\theta}$$

This is the speed necessary to create force balancing so no slippage occurs

some numbers

$R = 400$ ft
$\theta = 12°$
$g = 32$ ft/sec^2

$$v_R^2 = (400\text{ ft})(32\text{ ft/sec}^2)(\tan 12°) = 12800\left(\frac{\text{ft}}{\text{sec}}\right)^2\tan 12°$$

$$v_R = 52.2\ \frac{\text{ft}}{\text{sec}} \rightarrow \left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60^2\text{ sec}}{\text{hr}}\right) \quad \boxed{36.6\ \frac{\text{mi}}{\text{hr}}}$$

To increase this by 50% means (36.6 → 54.90 ≈ 55 mph)

$$v_{R_2} = 1.5\,v_R = 1.5\sqrt{Rg\tan 12°}$$

$$\sqrt{R_2 g\tan 12°}$$

square both sides

$$R_2 g\tan 12° = 1.5^2 Rg\tan 12°$$

$$\boxed{R_2 = 2.25R}$$

$$\boxed{= 900\text{ ft}}$$