Stewart Calculus 5e ET 12.1.42

reduction of a 3-space problem to Calc 2

To do: Find the volume of the solid that lies inside both of the spheres
$x^2+y^2+z^2+4x-2y+4z+5=0$ and $x^2+y^2+z^2=4$.

solution: complete squares on x, y, z

■ algebra

$(x+2)^2-4+(y-1)^2-1+(z+2)^2-4+5=0$

$(x+2)^2+(y-1)^2+(z+2)^2=4=2^2$

$C_1(-2,1,-2)$ center sphere 1

$r_1=2$ radius

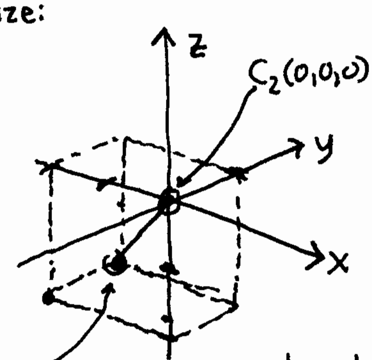
$C_2(0,0,0)$ center sphere 2

$r_2=2$ radius

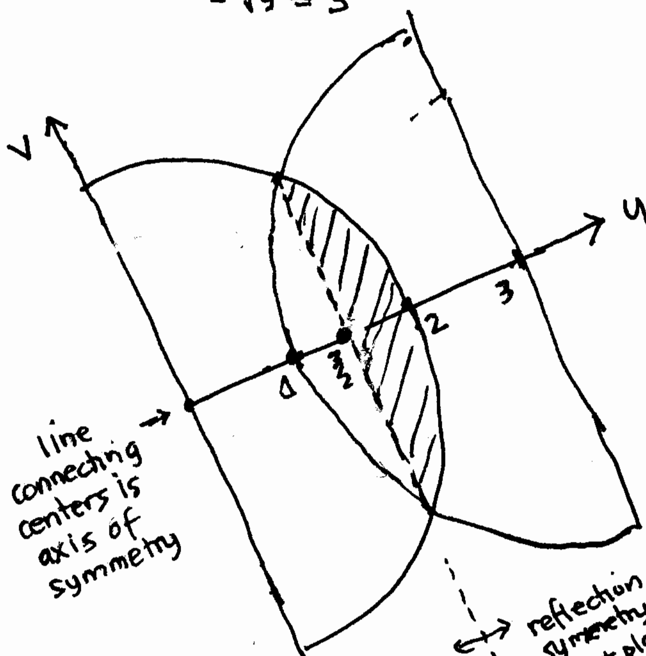
distance between centers

$d=((0-(-2))^2+(0-1)^2+(0-(-2))^2)^{1/2}$
$=\sqrt{9}=3$

■ visualize:



spheres too hard to draw (not for technology!)

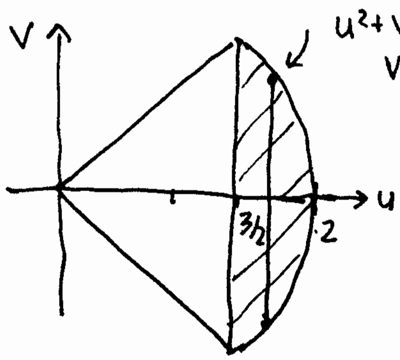


line connecting centers is axis of symmetry

reflection symmetry about plane equidistant from centers.

■ calc 2 application

By symmetry volume is twice volume of a "polar cap" of radius 2 and height 3/2



$V_{1/2}=\int_{3/2}^{2}\underbrace{\pi(4-u^2)}_{A(u)=\pi R(u)^2=\pi v^2}\,du \overset{\text{technology}}{=} \frac{11}{24}\pi$

cross-section area

$V=2V_{\frac{1}{2}}=\boxed{\frac{11}{12}\pi\approx 2.880}$ ratio ≈ 0.086

context: $V_{sphere}=\frac{4\pi}{3}2^3=\frac{32\pi}{3}\approx 33.511 \longrightarrow \sim 9\%$ of volume of either sphere