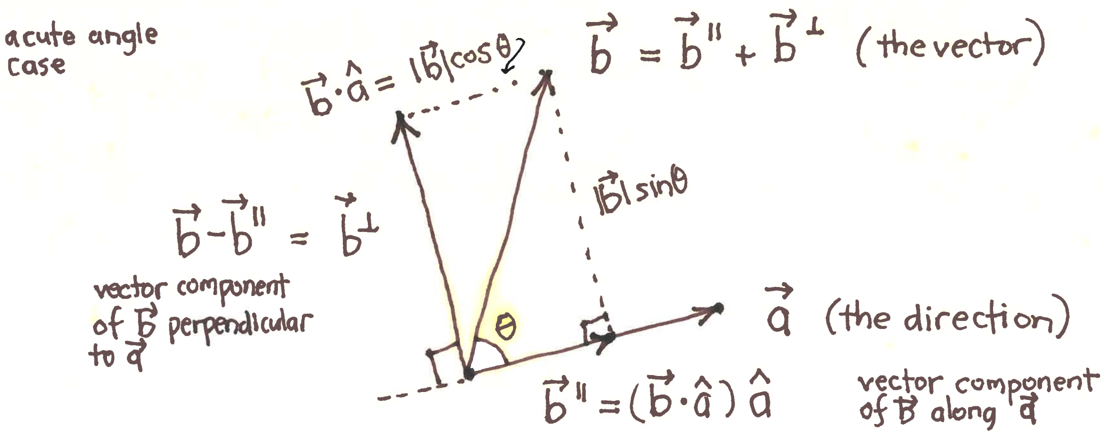
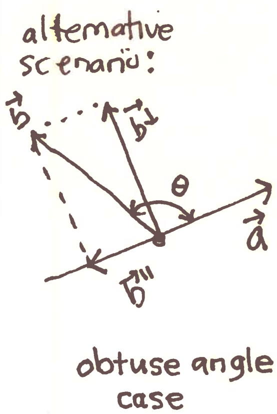
Decomposing a vector with respect to a direction





only the direction of $\vec{a}$ is relevant: $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$, $|\hat{a}| = 1$

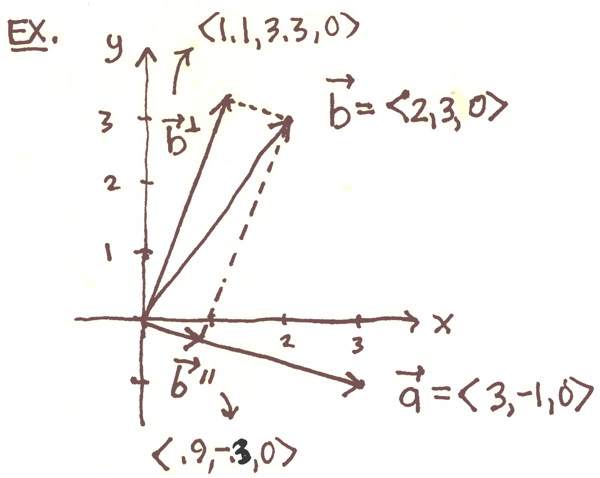
and this decomposes $\vec{b}$ into the sum of two orthogonal (perpendicular) vectors

Note:

$|\vec{b}|\cos\theta = |\hat{a}|\,|\vec{b}|\cos\theta = \hat{a}\cdot\vec{b}$ = scalar component of $\vec{b}$ along $\vec{a}$

$|\vec{b}|\sin\theta = |\hat{a}|\,|\vec{b}|\sin\theta = |\hat{a}\times\vec{b}|$ = scalar component of $\vec{b}$ perpendicular to $\vec{a}$

EX.



results consistent with hand graph

$|\vec{a}| = \sqrt{3^2+1^2+0^2} = \sqrt{10}$

$\hat{a} = \frac{1}{\sqrt{10}}\langle 3,-1,0\rangle$

$\vec{b}\cdot\hat{a} = \langle 2,3,0\rangle\cdot\dfrac{\langle 3,-1,0\rangle}{\sqrt{10}} = \dfrac{2\cdot 3+3(-1)}{\sqrt{10}} = \dfrac{3}{\sqrt{10}}$

$\vec{b}^{\parallel} = (\vec{b}\cdot\hat{a})\hat{a} = \frac{3}{\sqrt{10}}\frac{1}{\sqrt{10}}\langle 3,-1,0\rangle = \frac{3}{10}\langle 3,-1,0\rangle$

$= \langle \frac{9}{10}, -\frac{3}{10}, 0\rangle$

$\vec{b}^{\perp} = \vec{b}-\vec{b}^{\parallel} = \langle 2,3,0\rangle - \langle \frac{9}{10}, -\frac{3}{10}, 0\rangle = \langle \frac{11}{10}, \frac{33}{10}, 0\rangle$

aside

[note $|\vec{b}^{\perp}| = |\frac{11}{10}\langle 1,3,0\rangle| = \frac{11}{10}\sqrt{10}$

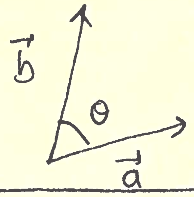
alternatively

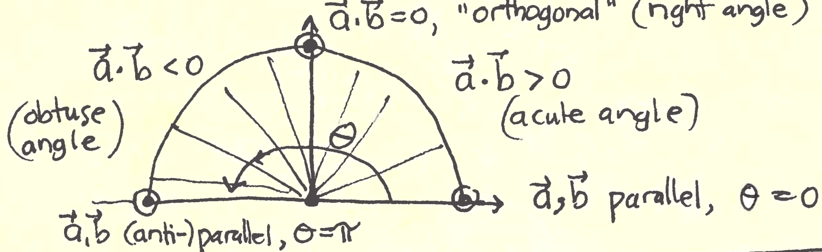
$\hat{a}\times\vec{b} = \frac{1}{\sqrt{10}}\vec{a}\times\vec{b} = \frac{1}{\sqrt{10}}\begin{vmatrix} i & j & k \\ 3 & -1 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \frac{1}{\sqrt{10}}\langle 0, 0, 9+2\rangle$

$= \frac{11}{\sqrt{10}}\langle 0,0,1\rangle$

$|\hat{a}\times\vec{b}| = \frac{11}{\sqrt{10}} = \frac{11\sqrt{10}}{10}$]

relative angle between 2 (nonzero) vectors





We can always choose the angle to be between 0 and π:

$\cos\theta = \hat{a}\cdot\hat{b}$

$\theta = \arccos(\hat{a}\cdot\hat{b})$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 0 \rightarrow |\vec{a}|=0$ or $|\vec{b}|=0$ or $\cos\theta = 0 \Leftrightarrow \theta=\frac{\pi}{2}$ (90°)

$\updownarrow$ $\qquad$ $\updownarrow$

$\vec{a}=0$ $\qquad$ $\vec{b}=0$

either $\vec{a}$ or $\vec{b}$ is zero (or both) — or — both are nonzero so an angle is defined and they are **"perpendicular" vectors**

"orthogonal vectors" have zero dot product

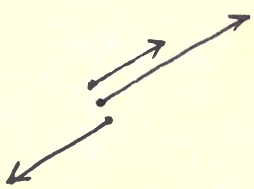
Remark: $|\vec{a}|=0$ is equivalent to $\vec{a}=0$

$\left(\sqrt{a_1^2+a_2^2+a_3^2} \rightarrow a_1=0=a_2=a_3 \rightarrow \vec{a}=\langle 0,0,0\rangle=\vec{0} \right)$

only the zero vector has zero length (not true in the geometry of the Lorentzian plane used in special relativity!!)

"parallel vectors" are proportional: $\vec{a}\propto\vec{b} \leftrightarrow \vec{a}=c\vec{b}$

(must be nonzero vectors to have a direction!)



How to tell?

example: $\langle 1,2,3\rangle$, $\langle 2,4,6\rangle$

$\langle 2,4,6\rangle \stackrel{?}{=} c\langle 1,2,3\rangle = \langle c,2c,3c\rangle$

$\downarrow$

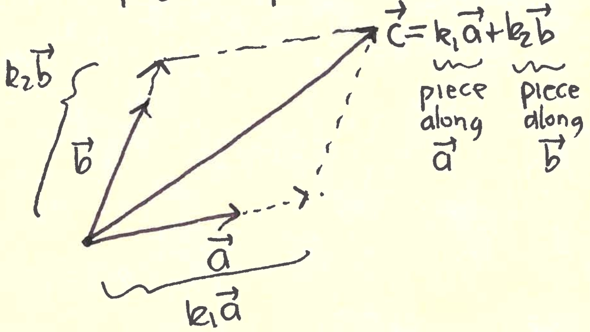
$2=c \rightarrow c=2$ ✓
$4=2c \rightarrow c=2$ ✓
$6=3c \rightarrow c=2$ ✓
} consistent soln. yes.

or ratios of corresponding components same:

$\frac{2}{1}=\frac{4}{2}=\frac{6}{3}=2=c$ ✓

PROJECTION PROCESSES

resolving a vector along 2 other vectors is a projection process (oblique projection)



with only 2 vectors, we can project one along the other using an orthogonal (= perpendicular) projection process, resolving one vector into a piece along the other (parallel to!) and a piece perpendicular to the other