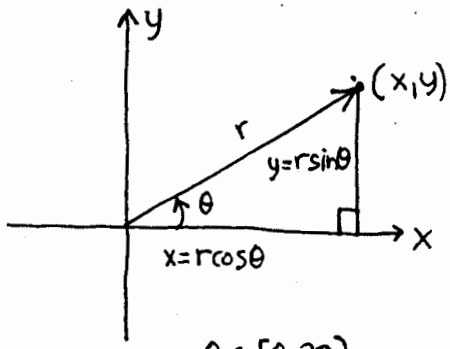
polar, cylindrical, spherical coordinates

polar coords in plane



$(x,y) = (r\cos\theta, r\sin\theta)$

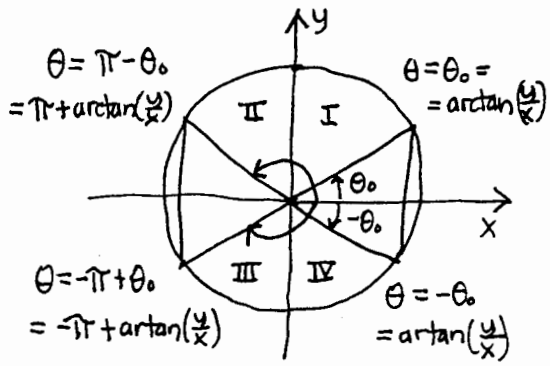
$x = r\cos\theta$
$y = r\sin\theta$

$\updownarrow$

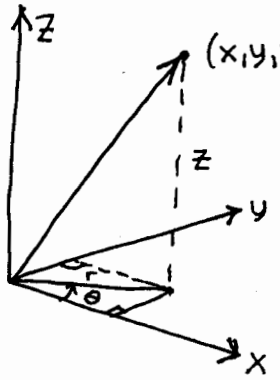
$r = \sqrt{x^2+y^2} \geq 0$

$\theta \in [0, 2\pi)$
or $\in (-\pi, \pi)$ $\quad \tan\theta = \frac{y}{x}$

MAPLE: $\theta = \arctan(y,x) \in (-\pi, \pi]$



cyl coords

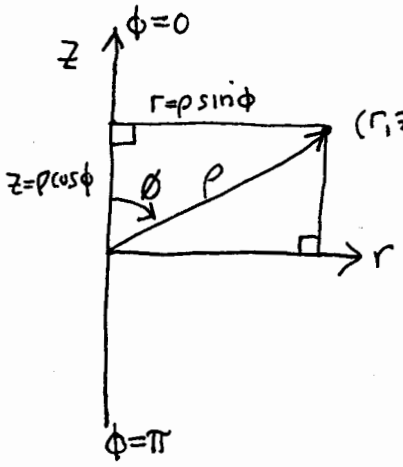


$(x,y,z) = (r\cos\theta, r\sin\theta, z)$

$x = r\cos\theta$
$y = r\sin\theta$ $\leftrightarrow$ $\tan\theta = \frac{y}{x}$
$z = z$

$r = \sqrt{x^2+y^2}$
$z = z$

cyl to sphere



$(r,z) = (\rho\sin\phi, \rho\cos\phi)$

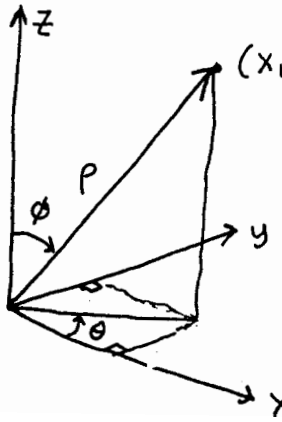
introduce polar coords in r-z plane (halfplane $r \geq 0$) of fixed θ but measured from the z-axis:

$0 \leq \phi \leq \pi$

$r = \rho\sin\phi$
$\theta = \theta$
$z = \rho\cos\phi$ $\leftrightarrow$

$\rho = \sqrt{r^2+z^2} \geq 0$
$\theta = \theta$
$\tan\phi = \frac{r}{z} \leftrightarrow \phi = \begin{cases} \arctan\left(\frac{r}{z}\right) \in [0, \frac{\pi}{2}), & z>0 \\ \pi + \arctan\left(\frac{r}{z}\right) \in (\frac{\pi}{2}, \pi], & z<0 \end{cases}$
$= \operatorname{arccot}\left(\frac{z}{r}\right) \in [0, \pi]$

sph coords



$(x,y,z) = (\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$

$x = \rho\sin\phi\cos\theta$
$y = \rho\sin\phi\sin\theta$ $\leftrightarrow$
$z = \rho\cos\phi$

$\rho = \sqrt{x^2+y^2+z^2}$
$\tan\phi = \sqrt{x^2+y^2}/z$
$\tan\theta = \frac{y}{x}$