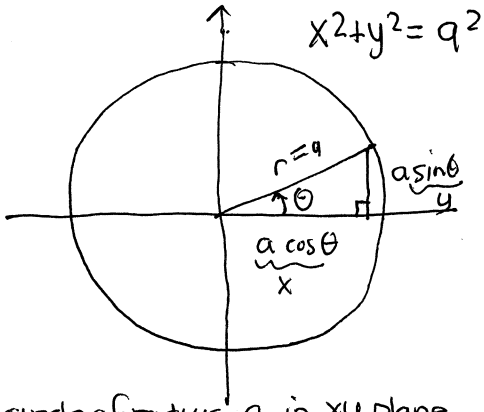
Curves & parametrized curves

a "curve" is a set of points along a "path"

a "parametrized curve" is a way of tracing out that path



circle of radius a in xy plane centered at origin

trace out by increasing polar angle from 0

let $\theta = t$ ("parameter")

by trigonometry:

$x = a\cos t,\ y = a\sin t$

choose $0 \le t \le 2\pi$ for one revolution counter-clockwise of circle.

eliminate parameter

$\frac{x}{a} = \cos t,\ \frac{y}{a} = \sin t$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = \cos^2 t + \sin^2 t = 1$

$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \rightarrow x^2+y^2=a^2$

variations: $0 \le t \le 4\pi$ → 2 revolutions of circle — trace out twice

clockwise? let $y \to -y$: $x = a\cos t,\ y = -a\sin t$ $\quad 0 \le t \le 2\pi$

[equivalently let $t \to -t$, same result]

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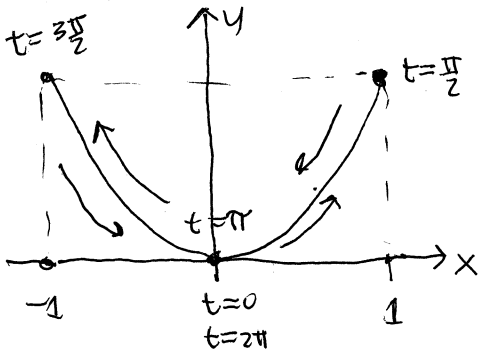
another example: $x = \sin t,\ y = \sin^2 t,\ 0 \le t \le 2\pi$

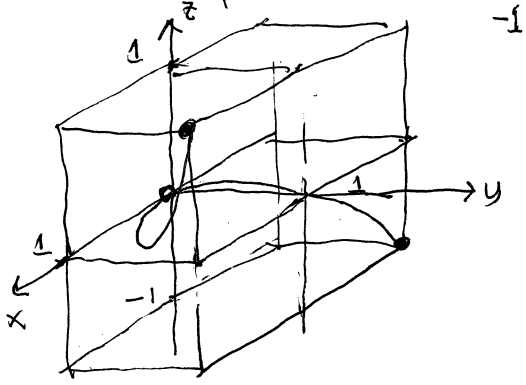
↳ eliminate parameter : $y = (\sin t)^2 = x^2$ but $|x| = |\sin t| \le 1$.

traces out segment of parabola twice.



3-d example: $x = t,\ y = t^2,\ z = t^3,$ "twisted cubic"

$-1 \le t \le 1$



look down z axis : $y = x^2$ parabola

look down y axis : $z = x^3$ cubic

look down x axis : $z = (\pm y^{1/2})^3 = \pm y^{3/2}$