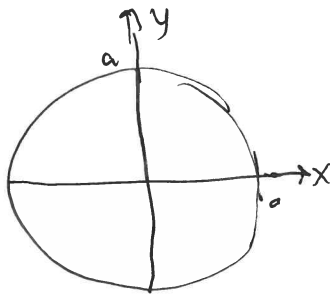
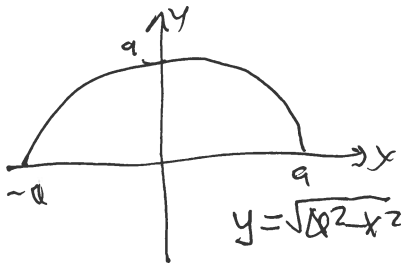


10.1 2d parametrized curves



$$x^2 + y^2 = a^2$$



curves defined by graphs or equations



parametrize?

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

$\underbrace{\hspace{1cm}}_{\cos^2 t} \quad \underbrace{\hspace{1cm}}_{\sin^2 t}$

$$\begin{aligned} \frac{x}{a} &= \cos t \rightarrow x = a \cos t \\ \frac{y}{a} &= \sin t \rightarrow y = a \sin t \end{aligned}$$

generalize to ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\begin{aligned} x &= a \cos t \\ y &= b \sin t \end{aligned}$$

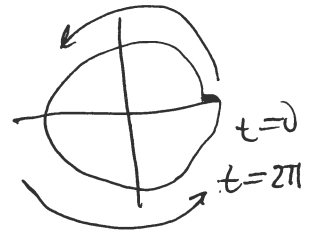


$$\theta = t$$

$$\begin{aligned} x &= a \cos \theta = a \cos t \\ y &= a \sin \theta = a \sin t \end{aligned}$$

$$t = 0 \dots 2\pi$$

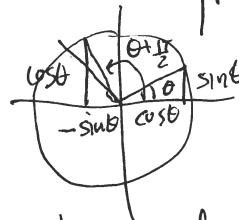
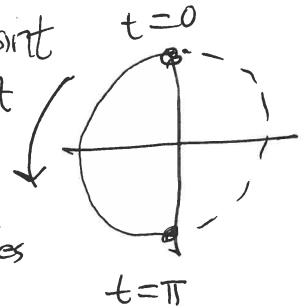
[flip $t \rightarrow -t$ to go opposite direction]



$$\begin{aligned} x &= a \cos\left(t + \frac{\pi}{2}\right) = -a \sin t \\ y &= a \sin\left(t + \frac{\pi}{2}\right) = a \cos t \end{aligned}$$

$$t = 0 \dots \pi$$

trig identities



parametrized curves trace out curves (paths) in a particular way.

eliminate parameter either by using identities or solving one equation for parameter and substituting into other equation

$$\begin{aligned} x^2 + y^2 &= (a \cos t)^2 + (a \sin t)^2 \\ &= a^2 \cos^2 t + a^2 \sin^2 t = a^2 (\cos^2 t + \sin^2 t) \\ &= a^2 \checkmark \end{aligned}$$

EX. $x = t^2$ $y = 2t \rightarrow t = \frac{y}{2}$

$$x = \left(\frac{y}{2}\right)^2 = \frac{1}{4}y^2$$

$x =$ function of y graph with y as ind var.