4D: \( \mathbb{R}^4 \rightarrow x \)

**df**, \( \frac{dy}{dx} \), ratio of differentials of \( x \) and \( y \) along tangent line to \( y = f(x) \)

**differential** = derivative times differentials of independent variable

2D: \( \mathbb{R}^2 \rightarrow x \)

**partial derivative**

\[ \frac{\partial}{\partial x} (...) = \frac{\partial f}{\partial x} \]

pronounce "partial \( x \) of (...)"

acts on expression to its right

containing variables \( x \) and \( y \)

(y held fixed during differentiation)

**vector derivative**

\[ \nabla = \left< \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right> = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} \]

pronounce "del"

\[ \nabla f = \left< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right> \]

produce vector field from function \( f \)

(\( \text{grad } f \))

**differential**

\[ df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = dz \]

differential = sum of partial derivatives times differentials of independent variables

\( dx, dy, dz \) are increments of variables along tangent plane