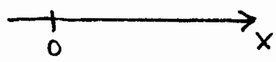
multivariable derivative and differential notation

1D: $\mathbb{R}$ 

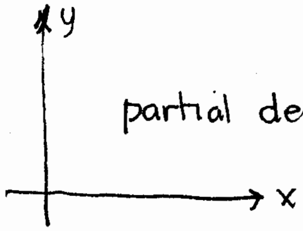
derivative $\frac{d}{dx}(\ldots)$ — acts on expression to its right containing variable x

pronounce "dee dee x" of (…) (= "the derivative wrt x of (…)")

$\frac{df}{dx} = \frac{dy}{dx}$ = ratio of differentials of x & y along tangent line to $y=f(x)$

$$df = \frac{df}{dx}\, dx$$

differential (dependent variable ⇔ function) = derivative times differential (independent variable)

2D: $\mathbb{R}^2$ 

partial derivative $\frac{\partial}{\partial x}(\ldots)$ — acts on expression to its right containing variables x & y (y held fixed during differentiation)

pronounce "partial x of (…)" (= "the partial derivative wrt x of (…)")

vector derivative $\vec{\nabla} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y}$ pronounce "del" ($\vec{\nabla} f \Leftrightarrow$ "del f")

$$\vec{\nabla}(\ldots) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle (\ldots) = \langle \frac{\partial}{\partial x}(\ldots), \frac{\partial}{\partial y}(\ldots) \rangle$$

$$= (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y})(\ldots) = \hat{\imath}\frac{\partial}{\partial x}(\ldots) + \hat{\jmath}\frac{\partial}{\partial y}(\ldots)$$

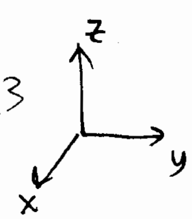
(…) = scalar expression in x,y to its right

$\vec{\nabla} f$ produces vector field from function f ("grad f")

differential $$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = dz$$

differential = sum of partial derivatives times differentials of independent variables

$dx, dy, df = dz$ are increments of variables along tangent plane

3D: $\mathbb{R}^3$ 

$$\vec{\nabla} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} f = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

$$= (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}) f = \hat{\imath}\frac{\partial f}{\partial x} + \hat{\jmath}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

notation acts like scalar multiplication on right of vector derivative

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$= \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle \cdot \langle dx, dy, dz \rangle = \vec{\nabla} f \cdot d\vec{r}$$

$\vec{r} = \langle x,y,z \rangle$, $d\vec{r} = d\langle x,y,z \rangle = \langle dx,dy,dz \rangle$

"del f dot dee r"