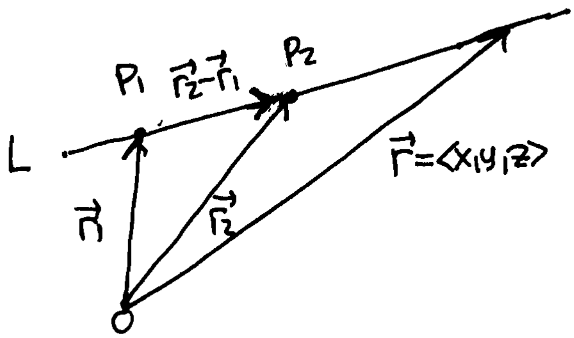
Describing lines and planes

2 distinct points determine a line



$\vec{a}=\langle a_1,a_2,a_3\rangle=\vec{r}_2-\vec{r}_1$ (or $\vec{r}_1-\vec{r}_2$)

$\vec{r}_0=\vec{r}_1$ (or $\vec{r}_2$)

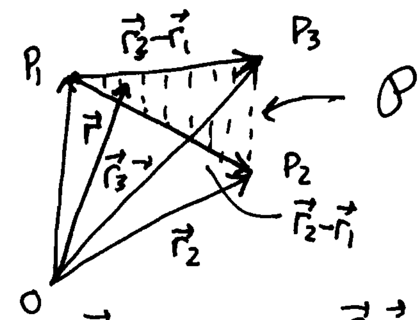
$$\boxed{\begin{aligned}\vec{r}&=\vec{r}_0+t\vec{a}\\ x&=x_0+ta_1\\ y&=y_0+ta_2\\ z&=z_0+ta_3\end{aligned}}$$

← line

parametrized equations of

plane →

3 distinct points not on a line determine a plane

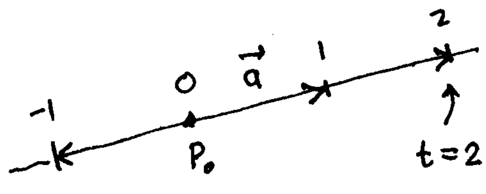


$\vec{a}=\langle a_1,a_2,a_3\rangle=\vec{r}_2-\vec{r}_1$ (or...)

$\vec{b}=\langle b_1,b_2,b_3\rangle=\vec{r}_3-\vec{r}_1$ (or...)

$\vec{r}_0=\vec{r}_1$ (or $\vec{r}_2$ or $\vec{r}_3$)

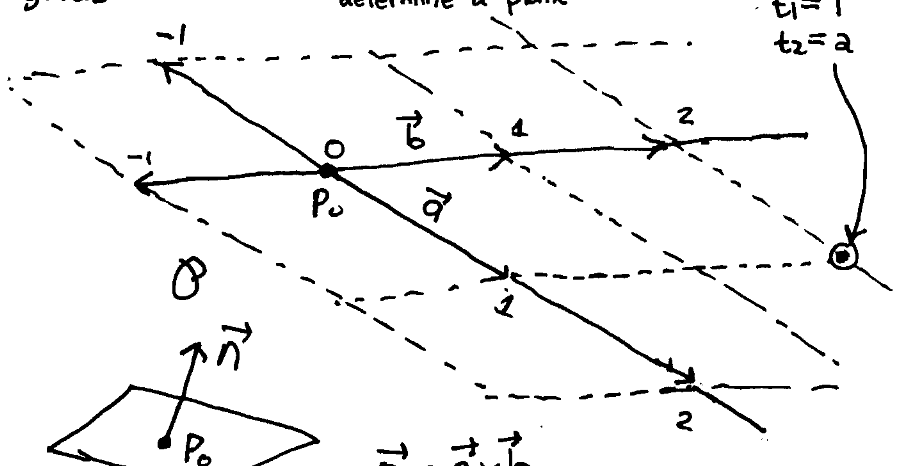
$$\boxed{\begin{aligned}\vec{r}&=\vec{r}_0+t_1\vec{a}+t_2\vec{b}\\ x&=x_0+t_1a_1+t_2b_1\\ y&=y_0+t_1a_2+t_2b_2\\ z&=z_0+t_1a_3+t_2b_3\end{aligned}}$$



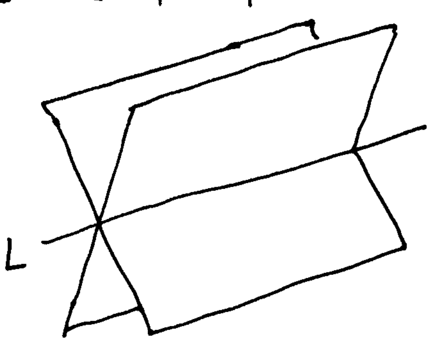
L

"parameter grids"

2 distinct intersecting lines determine a plane



2 nonparallel planes determine a line



$\vec{n}=\vec{a}\times\vec{b}$

$$\boxed{\begin{aligned}b_1x+b_2y+b_3z&=b_4\\ c_1x+c_2y+c_3z&=c_4\end{aligned}}$$

← line

unparametrized equations of

plane →

$$\boxed{\vec{n}\cdot(\vec{r}-\vec{r}_0)=0 \quad\text{or:}\quad n_1(x-x_0)+n_2(y-y_0)+n_3(z-z_0)=0}$$

The intersection of 2 nonparallel planes is a line. The solution of these 2 linear equations for x,y,z determines the coordinates of points on the line.

The above parametrized equations of the line are a way of representing the solution of these linear equations.

The equation of a plane is a linear equation to be solved for x,y,z to yield the coordinates of a point on the plane.

The above parametrized equations of the plane are a way of representing the solution of this linear equation.

aside:

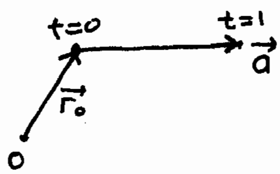
Parametrized line segments, parallelograms, parallelopipeds

Using parameter values between 0 and 1 we can easily describe uniquely all the points which belong to a line segment, a parallelogram, or a parallelopiped (interior and edges).

Let $\vec{a}, \vec{b}, \vec{c}$ be any 3 nonzero vectors which are not collinear or coplanar.

1-d: parametrized line segment

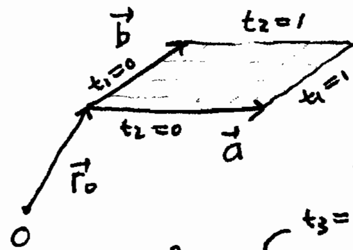
$$\vec{r} = \vec{r}_0 + t\,\vec{a}\,, \quad 0 \le t \le 1$$



endpoints

2-d: parametrized parallelogram

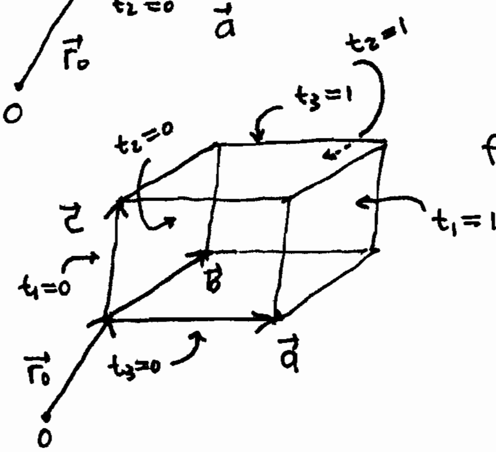
$$\vec{r} = \vec{r}_0 + t_1\vec{a} + t_2\vec{b}\,, \quad 0 \le t_1 \le 1, \quad 0 \le t_2 \le 1$$



edges

3-d: parametrized parallelopiped

$$\vec{r} = \vec{r}_0 + t_1\vec{a} + t_2\vec{b} + t_3\vec{c}\,, \quad 0 \le t_1 \le 1, \quad 0 \le t_2 \le 1, \quad 0 \le t_3 \le 1$$



faces

Thus a single idea works in all 3 dimensions within space. This is the unity of mathematics. If this does not matter to you, remember at least the first example.

→ We need this later: store in memory.