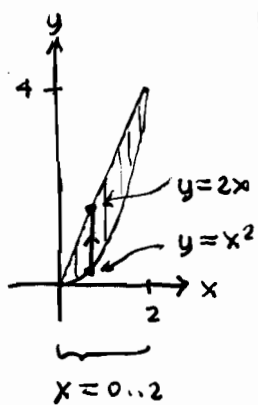


double integrals : it's really about describing a region of the plane



■ Consider the region R between the graphs $y=x^2$ and $y=2x$.

These curves intersect at : $x^2=2x$ or $0=x^2-2x=x(x-2) \rightarrow x=0,2$

$\rightarrow y=0,4$

points (0,0) and (0,4).

The diagram shows a "typical" vertical cross-section and its "direction" (increasing y) with endpoints labeled by start/stop values

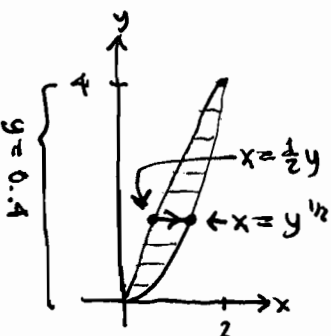
$$R: y = x^2 \dots 2x \text{ (increasing bot. to top)}$$

$$\text{as } x = 0 \dots 2 \text{ (increasing from left to right)}$$

This describes the region as a "type I" region (y first, then x)

$$\iint_R f(x,y) dA = \int_{x=0}^{x=2} \int_{y=x^2}^{y=2x} f(x,y) dy dx = \int_0^2 \int_{x^2}^{2x} f(x,y) dy dx$$

\downarrow
 $y = x^2 \dots 2x$ as $x = 0 \dots 2$



■ But we can also describe the bounding curves with x expressed as a function of y, a "type II" region (x first, then y)

$$y = x^2 \xrightarrow{x \geq 0} x = y^{1/2}$$

$$y = 2x \rightarrow x = \frac{1}{2}y$$

$$R: x = \frac{1}{2}y \dots y^{1/2} \text{ (increasing left to right)}$$

$$\text{as } y = 0 \dots 4 \text{ (increasing bot. to top)}$$

The diagram shows a "typical" horizontal cross-section and its "direction" (increasing x) with endpoints labeled by start/stop values

$$\iint_R f(x,y) dA = \int_{y=0}^{y=4} \int_{x=\frac{1}{2}y}^{x=y^{1/2}} f(x,y) dx dy = \int_0^4 \int_{\frac{1}{2}y}^{y^{1/2}} f(x,y) dx dy$$

\downarrow
 $x = \frac{1}{2}y \dots y^{1/2}$ as $y = 0 \dots 4$

technique : changing the order of integration

For a region that allows either choice above, we can start with one order of integration, make a diagram of the region of integration and its bounding curves, then re-express them as above and determine the new limits of integration

$$\int_0^2 \int_{x^2}^{2x} f(x,y) dy dx = \int_{x=0}^{x=2} \int_{y=x^2}^{y=2x} f(x,y) dy dx \rightarrow \text{make diagram} \rightarrow \text{re-describe}$$

\leftarrow then go backwards to the new double integral

$$z = f(x,y)$$

helps visualize integral in 3D as signed volume between graph and xy plane.

integrand is function whose graph in 3D leads to solid associated with 2D integral

REMARK