**Green's Theorem:**

\[ \int_C F \cdot T \, ds = \int_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA \]

**Gauss and Stokes**

Let \( N \) be the outer unit normal (points away from \( D \)) obtained from the unit tangent \( T \) by a 90° clockwise rotation:

\[ T = \langle T_1, T_2 \rangle \rightarrow N = \langle N_1, N_2 \rangle = \langle -T_2, -T_1 \rangle \]

Any vector field in the plane can be rotated 90° counterclockwise:

\[ P = \langle P_1, P_2 \rangle \rightarrow G = \langle G_1, G_2 \rangle = \langle P_2, -P_1 \rangle \]

\[ F_n = P \times T = G \cdot N = G_1 \]

Since rotation does not change the dot product between two vectors:

\[ \langle \vec{G}, \vec{N} \rangle = N_1G_1 + N_2G_2 = T_1G_1 + (-T_2)G_2 = T_1G_1 + T_2G_2 = \hat{F} \cdot \hat{P} \]

So, tangential component of \( F \) equals the (outer) normal component of \( G \) (colors can be positive, negative, or zero).

**Stokes' Theorem version:**

\[ \int_C F \cdot \hat{T} \, ds = \oint \text{curl} F \cdot \hat{N} \, dA \]

**Gauss' Law version:**

\[ \int_C \vec{E} \cdot \hat{N} \, dA = \oint \vec{D} \cdot d\vec{S} = \oint \text{div} \vec{G} \, dA \]