divergence and curl

$$\vec{F} = \langle F_1, F_2, F_3 \rangle$$

$$\vec{\nabla} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\vec{\nabla} \cdot \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle F_1, F_2, F_3 \rangle = \boxed{\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = \operatorname{div} \vec{F}}$$

$$\vec{\nabla} \times \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle F_1, F_2, F_3 \rangle$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \boxed{\left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle = \operatorname{curl} \vec{F}}$$

$$\hat{k} \cdot \operatorname{curl} \vec{F} = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

If $F_3 = 0$ $\quad \operatorname{div} \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}$

combinations which enter Greens Theorem for 2-d vector fields

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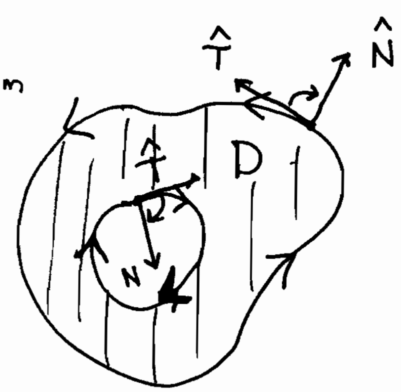
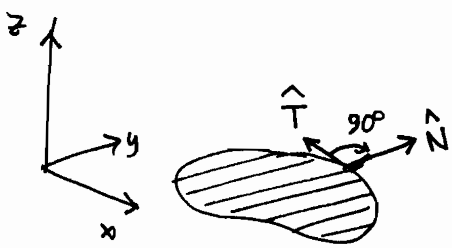
Right Hand Rule : Thumbs Down!

2d Greens Thm with hole



$\hat{T}$ and outer $\hat{N}$ always related by clockwise $90°$ rotation

Within 3d space, $F_3 = 0$ 2d vector:



$\vec{F} \to -\hat{k} \times \vec{F}$ performs clockwise rotation in x-y plane

$$-\hat{k} \times \vec{F} = \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ F_1 & F_2 & 0 \end{vmatrix} = \langle F_2, -F_1, 0 \rangle$$

thumb points down in space, fingers curl in the clockwise direction in x-y plane