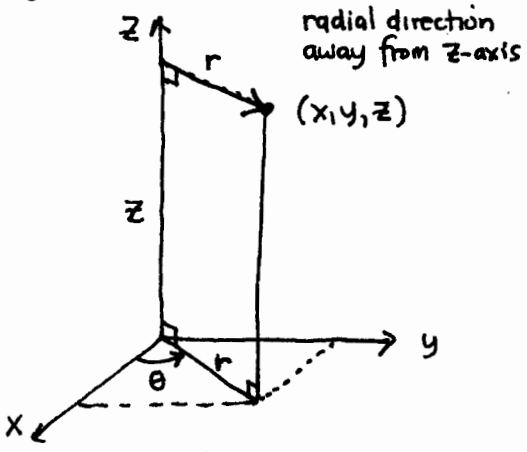
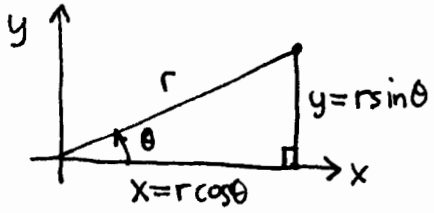
cylindrical coordinates (r, θ, z)



radial direction away from z-axis



$r \geq 0$, $0 \leq \theta \leq 2\pi$ (or $-\pi \leq \theta \leq \pi$)

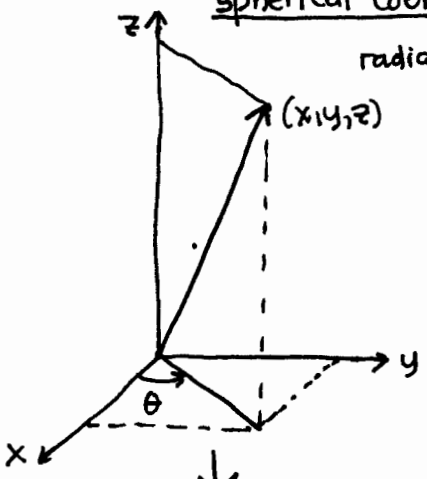
→ keep z, polar coords in xy plane

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

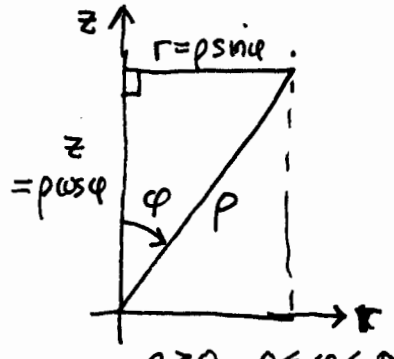
$x^2+y^2 = r^2 \rightarrow r = \sqrt{x^2+y^2} \geq 0$

$\frac{y}{x} = \tan\theta \rightarrow \theta = \arctan\frac{y}{x} + \begin{cases} 0; & \text{I, IV} \\ \pi; & \text{II} \\ -\pi; & \text{III} \end{cases}$ (quad)

spherical coordinates (ρ, φ, θ)



radial direction away from origin



$\rho \geq 0$, $0 \leq \varphi \leq \pi$ (up) (down)

dont memorize just remember diagram

$$r = \rho\sin\varphi$$
$$z = \rho\cos\varphi$$

→ keep θ, polar coords in rz plane

$$x = (\rho\sin\varphi)\cos\theta$$
$$y = (\rho\sin\varphi)\sin\theta$$
$$z = (\rho\cos\varphi)$$

$x^2+y^2+z^2 = \rho^2 \rightarrow \rho = \sqrt{x^2+y^2+z^2} = \sqrt{r^2+z^2}$

$\cos\varphi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\varphi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

Example. Find cyl/sph coords of $(x,y,z) = (-1, 2, 1)$.

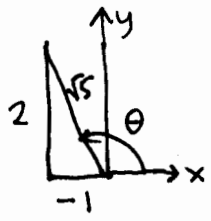
visualize

think, use simple trig

$(-1, 2, 1) \rightarrow \rho = \sqrt{1+4+1} = \sqrt{6}$

or

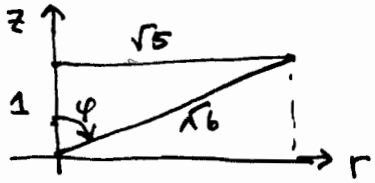
$r = \sqrt{1+4} = \sqrt{5} \rightarrow z = 1 \rightarrow \rho = \sqrt{5+1} = \sqrt{6}$



$\tan\theta = \frac{2}{-1}$

$\theta = \pi - \arctan 2$

$(\approx 116.6°)$



$\cos\varphi = \frac{1}{\sqrt{6}}$

$\varphi = \arccos\frac{1}{\sqrt{6}}$

$(\approx 65.9°)$