2D max-min 2^{nd} derivative test

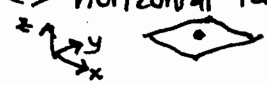
1D :

derivative icons

- horizontal tangent line at a point $f' = 0$
- concave up (happy) positive $f'' > 0$
- concave down (sad) negative $f'' < 0$

2nd derivative test where $f' = 0$	
$f'' > 0$	local min
$f'' < 0$	local max
$f'' = 0$	inconclusive

2D : critical point where $f_x(x_0, y_0) = 0 = f_y(x_0, y_0)$ $\Leftrightarrow$ horizontal tangent plane



local min  , local max 

chart of signs of 2nd derivatives at (x_0, y_0) :

f_{xx}	+ } local min?	− } local max?	+ } saddle!	− } saddle!	0! or 0?
f_{yy}	+	−	−	+	
$f_{xx}f_{yy} - f_{xy}^2$	+ confirm − saddle 0 inconclusive	+ confirm − saddle 0 inconclusive	−	−	0 or −

$f_{xx} f_{yy} < 0$ already
$f_{xx}f_{yy} - f_{xy}^2 < 0$ automatic

If f_{xx}, f_{yy} have same sign then $f_{xx} f_{yy} > 0$.
If still > 0 when subtract f_{xy}^2, then initial guess is confirmed.

when < 0, then in some directions between the axes, the concavity is reversed, so saddle.

$f_{xx} f_{yy} = 0$
$f_{xx}f_{yy} - f_{xy}^2 \le 0$
same 2 conclusions