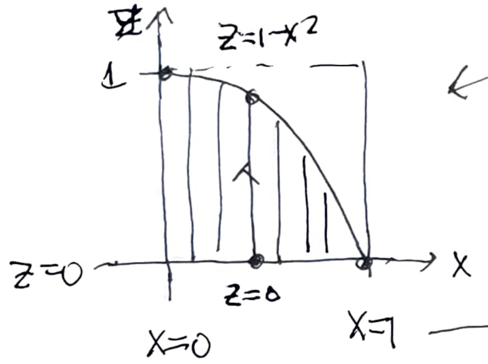
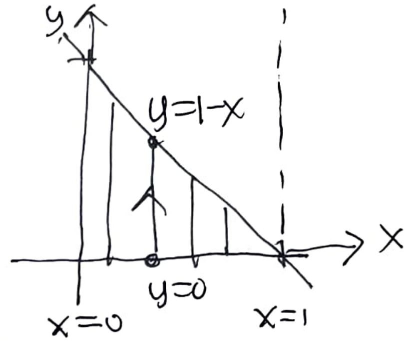
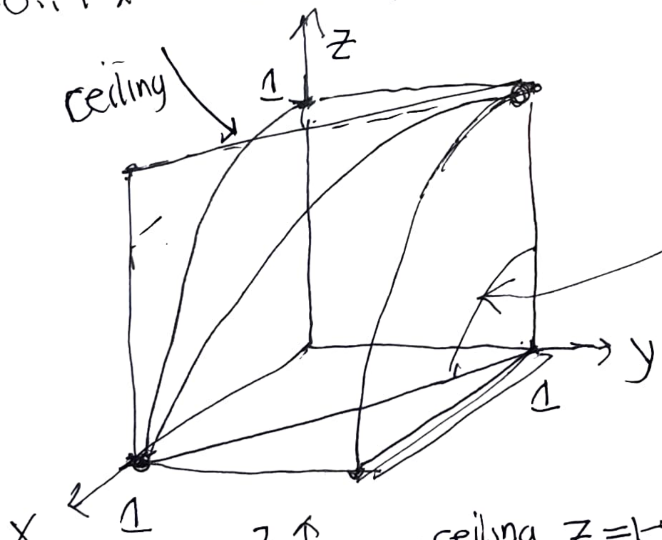
triple integral deconstruction (1)

$$Q=\int_0^1\int_0^{1-x^2}\int_0^{1-x} f\,dy\,dz\,dx \rightarrow \underbrace{\int_{x=0}^{x=1}\int_{z=0}^{z=1-x^2}}\ \underbrace{\int_{y=0}^{y=1-x}}\ f\,dy\,dz\,dx$$

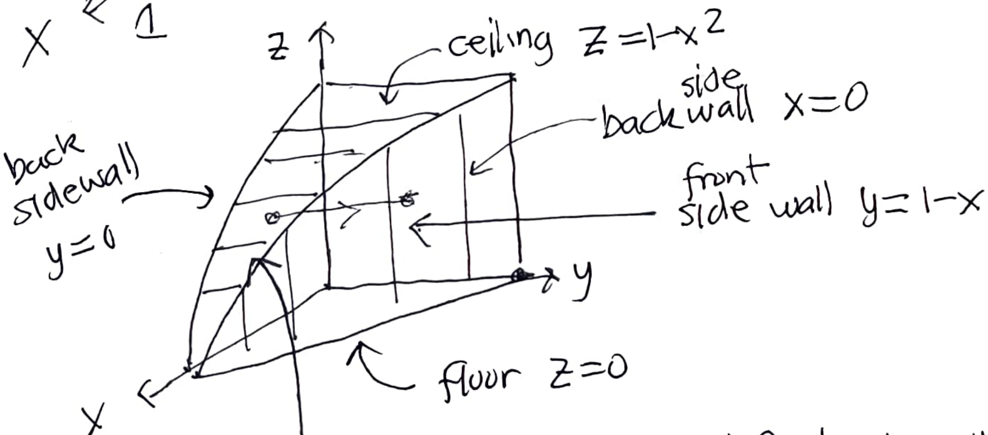




$z=0..1-x^2$ while $x=0..1$



vertical side wall must intersect ceiling in curve above $y=x^2$ between $(1,0,0)$ and $(0,1,1)$

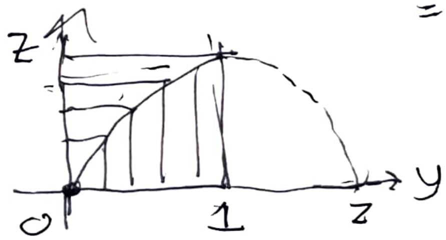


y-first diagram

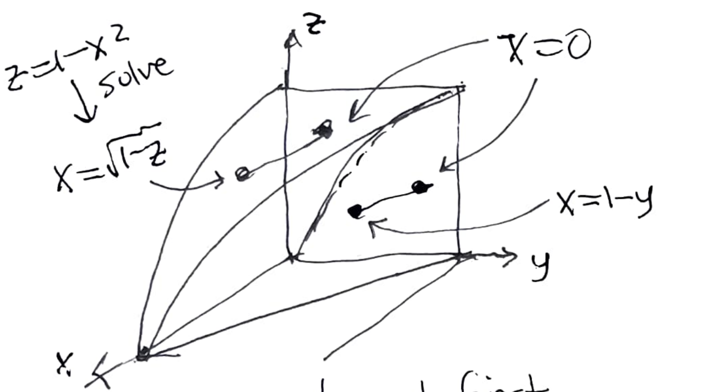
intersection of ceiling and front sidewall:

$\left.\begin{array}{l} z=1-x^2 \\ y=1-x \end{array}\right\} \rightarrow$ eliminate x : $x=1-y \rightarrow z=1-(1-y)^2$
$= 2y-y^2 = (2-y)y$

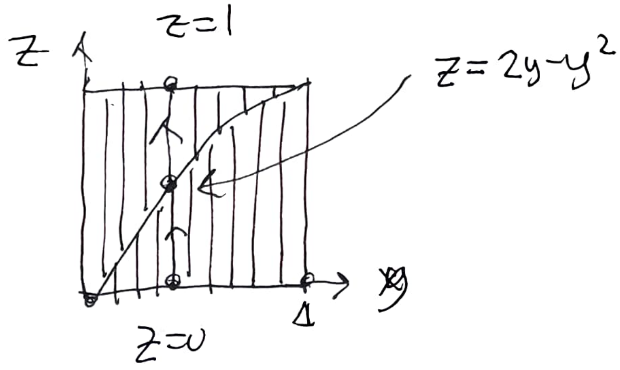
looking down x-axis
2 ceiling expressions, floor $x=0$
need sum of 2 triple integrals



triple integral deconstruction (2)



do x integral first
(2 separate triple integrals)

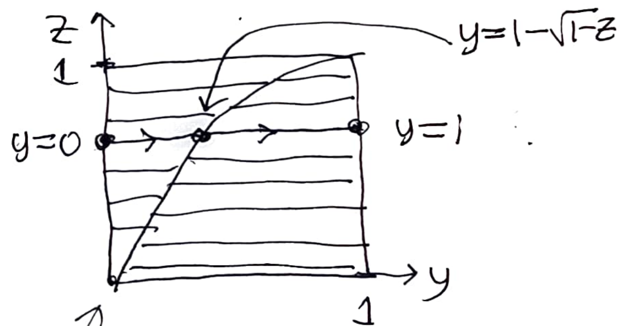


below: $x=0..1-y$
while $z=0..2y-y^2$
while $y=0..1$

top: $x=0..\sqrt{1-z}$
while $z=2y-y^2..1$
while $y=0..1$

$$Q=\int_0^1\int_0^{2y-y^2}\int_0^{1-y} f\,dx\,dz\,dy + \int_0^1\int_{2y-y^2}^{1}\int_0^{\sqrt{1-z}} f\,dx\,dz\,dy$$

or



$z=2y-y^2 \to y^2-2y+z=0$

$y=\dfrac{2\pm\sqrt{4-4(1)z}}{2}$

$=1\pm\sqrt{1-z}$

root is ≤ 1

intersection (inverted)

below (left): $x=0..\sqrt{1-z}$
while $y=0..1-\sqrt{1-z}$
while $z=0..1$

above (right): $x=0..1-y$
while $y=1-\sqrt{1-z}..1$
while $z=0..1$

$$Q=\int_0^1\int_0^{1-\sqrt{1-z}}\int_0^{\sqrt{1-z}} f\,dx\,dy\,dz + \int_0^1\int_{1-\sqrt{1-z}}^{1}\int_0^{1-y} f\,dx\,dz\,dy$$

<u>exercise</u>. do 2+1 diagrams for z-first iterations & rewrite the integral