Before we move on from double integrals, one useful fact remains which applies to triple integrals as well.

Consider an integrand which factors into a product of the independent variables, with $\boxed{\text{constant}}$ limits of integration.

$$\int_a^b \int_c^d \underbrace{f(x,y)}_{} \, dy\, dx = \int_a^b \int_c^d \underbrace{F(x)}_{\text{ind. of } y!} G(y)\, dy\, dx$$

$$= F(x)\, G(y) \qquad = \int_a^b F(x) \underbrace{\int_c^d G(y)\, dy}_{\text{constant!}} \, dx$$

$$= \left(\int_c^d G(y)\,dy\right)\left(\int_a^b F(x)\, dx\right)$$

> The integral of a "factorizable integrand" with constant limits of integration factors into the separate integrals of the factor functions.

example

$$\int_0^{\frac{\pi}{2}} \int_0^a (r^2 \cos^2\theta)\, r\, dr\, d\theta = \underbrace{\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta}_{\text{Maple}} \ \underbrace{\int_0^a r^3\, dr}_{\text{easy}}$$

This often occurs with rotational symmetry or in cylindrical & spherical symmetry in 3 dimensions.