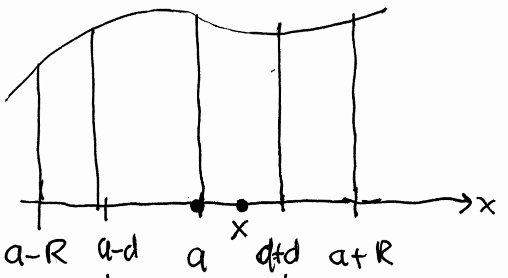

error estimate for series here $|x-a|<d$

inside interval of convergence

$$f(x) = \underbrace{f(a) + f'(a)(x-a)}_{T_1(x)} + \underbrace{\tfrac{1}{2} f''(a)(x-a)^2 + \cdots}_{R_1(x)}$$

assume $|f''(x)| \le M$ for $|x-a| \le d$

assume $x > a$ for derivation

$$\underbrace{\int_a^x f''(t)\,dt}_{f'(t)\big|_a^x = f'(x) - f'(a)} \le \underbrace{\int_a^x M\,dt}_{M(x-a)}$$

$$f'(x) \le f'(a) + M(x-a)$$

$$\underbrace{\int_a^x f'(t)\,dt}_{f(t)\big|_a^x = f(x) - f(a)} \le \underbrace{\int_a^x f'(a) + M(t-a)\,dt}_{} $$

$$f(t)\Big|_a^x = f(x) - f(a) \le f'(a)t + \frac{M}{2}(t-a)^2\Big|_a^x = f'(a)(x-a) + \frac{M}{2}(x-a)^2$$

$$f(x) \le \underbrace{f(a) + f'(a)(x-a)}_{T_1(x)} + \frac{M}{2}(x-a)^2$$

$$R_1(x) = f(x) - T_1(x) \le \frac{M}{2}(x-a)^2$$

repeat with $f''(x) > -M$

find $-\frac{M}{2}(x-a)^2 \le R_1(x)$

conclude $|R_1(x)| \le \frac{M}{2}|x-a|^2 \longrightarrow$ extend to $x < a$

THEN ITERATE $|R_2(x)| \le \frac{M}{3!}|x-a|^3$ integrate 3 times instead of twice

...

$$\boxed{|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}}$$ for $|x-a| \le d < R$

where $\boxed{|f^{(n+1)}(x)| \le M}$

This gives us tool to prove $\lim_{n\to\infty} R_n(x) = 0$ for Taylor series

and gives estimate to guarantee precision of approximation by truncating the series.

(see e^x example Maple worksheet)