Plotting a simple functions with parameters

$y = h\left(\frac{x^2}{R^2} - 1\right)$ on $0 \le x \le R$. How to plot this on the given interval?

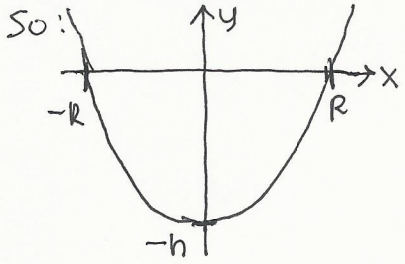
The problem deals with a parabolic tank having two physical length dimensions: a height h and a radius R. Physical dimensions are always positive, $h > 0$, $R > 0$. And it should hold liquid! If $h < 0$ it is upside down.

This is an even quadratic function so its graph is a parabola with its vertex on the y-axis. The coefficient of x^2 is positive so it opens upwards.

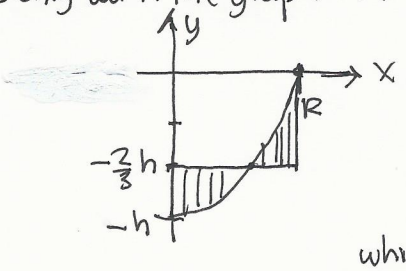
The y-intercept occurs at $x = 0$: $y = h(0^2/R^2 - 1) = -h$.

The x-intercepts have $y = 0 = h\left(\frac{x^2}{R^2} - 1\right) \rightarrow \frac{x^2}{R^2} - 1 = 0 \rightarrow x^2 = R^2 \rightarrow x = \pm R$

So:



But we only want the graph on the interval $0 \le x \le R$



We find $y_{avg} = -\frac{2}{3}h$ so to illustrate its meaning, draw the square which should have the same area as above the curve.

Thus the two shaded regions should have the same areas.

Note $\frac{y}{h} = \left(\frac{x}{R}\right)^2 - 1 \xrightarrow[\frac{y}{h} = Y]{\frac{x}{R} = X} Y = X^2 - 1$.

X, Y are dimensionless variables. The new graph is equivalent to setting $h = R = 1$, or measuring x and y in units of R and h respectively.

You can plot this curve with technology.