Stewart 8e 7.8.62 : ideal gas speed distribution

$$\bar{v} = \frac{4}{\sqrt{\pi}} \left(\frac{M}{2RT}\right)^{3/2} \int_0^\infty v^3 e^{-\frac{Mv^2}{2RT}}\, dv$$

M = molecular weight of gas
R is the gas constant
T is the gas temperature
v is the molecular speed
$\bar{v}$ is the average speed of molecules in an ideal gas

$$\frac{Mv^2}{2RT} = \frac{v^2}{\left(\frac{2RT}{M}\right)} \equiv \frac{v^2}{V^2} \quad \text{where } V = \left(\frac{2RT}{M}\right)^{1/2}$$

must have dimensions of speed since the exponential must have an input that does not depend on the choice of units!

Therefore $u = \frac{v}{V}$ is a dimensionless speed variable measuring speed in multiples of the velocity constant V which must characterize the velocity behavior of this system.
since there are no other parameters which can interfere, the average speed must be a "pure number" times V.

change of variable: $u = \frac{v}{V}$ $\quad du = \frac{dv}{V}$ $\rightarrow$ $v = Vu$, $dv = V\,du$, $v=0: u=0$, $v \to \infty : u \to \infty$

both speed and differential of speed "scale" under this change of variable. which acts like a unit conversion factor (inches to cm, ft-lbs to N-m, etc)

$$\bar{v} = \frac{4}{\sqrt{\pi}} V^{-3} \int_0^\infty (uV)^3 e^{-u^2} (V\,du) = \frac{4}{\sqrt{\pi}} V^{3+1-3} \left(\int_0^\infty u^3 e^{-u^2} du\right)$$

$$= \frac{4}{\sqrt{\pi}} V \underbrace{\left(\int_0^\infty u^3 e^{-u^2} du\right)}_{\text{pure number} = \frac{1}{2}}$$

carries speed units since result must be a speed.

$$\bar{v} = \frac{4}{\sqrt{\pi}} V \cdot \frac{1}{2} = \frac{2}{\sqrt{\pi}} V \approx 1.128$$

or $\frac{\bar{v}}{V} = \frac{2}{\sqrt{\pi}} \approx 1.128$

The average speed comes out equal to the "characteristic" speed V modulo a geometrical or numerical factor "of order unity"

Dimensional analysis is very important in applications.
Problem 7.8.66 on "stellar stereograph," is another good example where the distribution of mass in a star (or globular cluster) of radius R benefits from introducing a dimensionless radial variable $u = r/R$ where r is the actual radius of a location in the star from its center
Instead of using a "dummy radial variable" in the integral, it introduces a copy variable s, so $\mathcal{S} = s/R$ is also appropriate, with
$0 \le r \le R \leftrightarrow \quad 0 \le u \le R, \quad 0 \le \mathcal{S} \le R.$

$$y(s) = \int_s^R \frac{2r}{\sqrt{r^2 - s^2}} x(r)\, dr \longrightarrow y(r) = \int_r^R \frac{2t}{\sqrt{t^2 - r^2}} x(t)\, dt$$ "dummy variable t"

same radial variable

can use its symbol here if we introduce