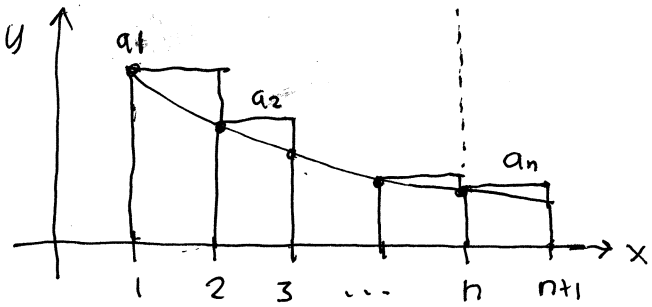
infinite series integral test via diagrams

$$a_n = f(x) > 0 \text{ on } 1 \le x \le \infty, \text{ decreasing, with } \lim_{x\to\infty} f(x) = 0.$$

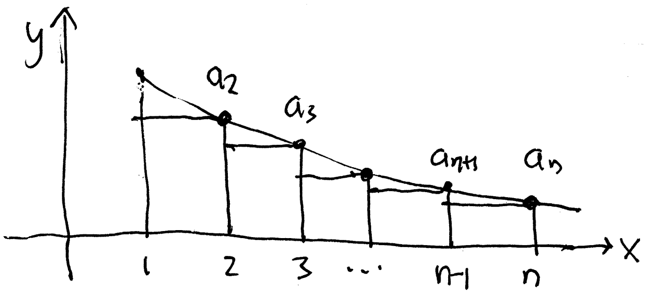
$$S = \sum_{n=1}^{\infty} a_n$$



$$S_n = \sum_{k=1}^{n} a_k = \text{left endpoint Riemann sum for } 1 \le x \le n+1 \text{ and } \Delta x = 1$$

$$> \int_1^{n+1} f(x)\,dx \xrightarrow{n\to\infty} \int_1^{\infty} f(x)\,dx$$

if diverges then S_n must diverge so S diverges



$$S_n - a_1 = \sum_{k=2}^{n} a_k = \text{right endpoint Riemann sum for } 1 \le x \le n \text{ and } \Delta x = 1$$

$$< \int_1^n f(x)\,dx \quad \text{so}$$

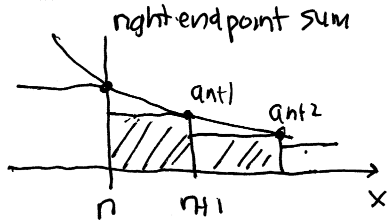
$$S_n < a_1 + \int_1^n f(x)\,dx \xrightarrow{n\to\infty} a_1 + \int_1^{\infty} f(x)\,dx$$

if converges then S_n must converge so S converges

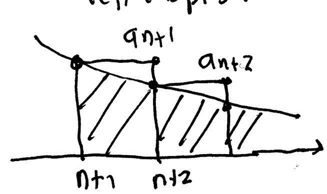
now focus on remainder.

$$S = S_n + \underbrace{R_n}_{\text{rest of series} \;=\; \text{error if truncating series at } n} \quad : \quad R_n = \sum_{k=n+1}^{\infty} a_n$$

right endpoint sum



left endpt sum



$$\int_n^{\infty} f(x)\,dx > \sum_{k=n+1}^{\infty} a_k = R_n > \int_{n+1}^{\infty} f(x)\,dx$$

so

$$\underbrace{\int_{n+1}^{\infty} f(x)\,dx}_{R_{n,\text{left}}} < R_n < \underbrace{\int_n^{\infty} f(x)\,dx}_{R_{n,\text{right}}}$$

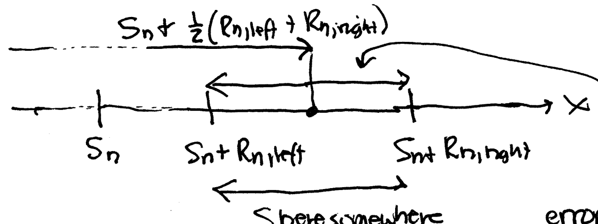
($R_{n,\text{right}}$: maximum error) use to determine n for maximum error less than...

now add S_n to each member of inequality.

$$S_n + R_{n,\text{left}} < \underbrace{S_n + R_n}_{=S} < S_n + R_{n,\text{right}}$$

puts error bars around unknown value S

$S_n + \frac{1}{2}(R_{n,\text{left}} + R_{n,\text{right}})$



error bar radius:

$$= \frac{1}{2}(R_{n,\text{right}} - R_{n,\text{left}}) > \text{error if take } \boxed{S \approx S_n + \frac{1}{2}(R_{n,\text{left}} + R_{n,\text{right}})}$$