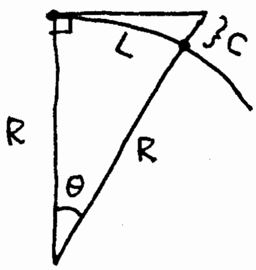
Taylor Series in Real Life: Worksheet

[Stewart Calculus 5e, 11.12.35]



If a surveyor measures differences in elevation when making plans for a highway across a flat desert, corrections must be made for the curvature of the Earth. Let R be the mean radius of the Earth (at sea level) and let L be the length of the highway, assumed to be a circular arc at the surface of the Earth. $R = 6371$ km (Google).

a) Show that the correction C is given by $C = R\left(\sec\left(\frac{L}{R}\right) - 1\right)$.
[Hint: $C+R$ is the hypotenuse!]

b) By polynomial long division find the first 3 nonzero terms in the Taylor series for $\sec(x) = \dfrac{1}{\cos x} = \dfrac{1}{1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots}$. [Check with Maple's taylor command.]

c) Then evaluate the first 2 nonzero terms in the Taylor series for $\sec(x) - 1$.

d) Now letting $x = L/R$ in this result, show that $C \approx R\left(\dfrac{L^2}{2R^2} + \dfrac{5L^4}{24R^4}\right)$.

e) If the highway were approximately at sea level we can use $R \approx 6371$ km. For $L = 100$ km, evaluate $x = L/R$ and evaluate the two contributions $C = C_1 + C_2$ to the correction separately, giving the first term C_1 in meters and the second term C_2 in centimeters. How do these compare to a direct evaluation of part a) with technology?

f) Suppose we try to apply this to the 40 mile straightarrow stretch of I-80 crossing the Bonneville Salt Flats east of Salt Lake City, Utah: elevation 4218 ft (from Google). Convert this added elevation to meters and add it to R to obtain the new value appropriate to this scenario. Then using $L = 40$ mi (how many km?), recalculate the new values of $x = R/L$ and C_1 and C_2, again in meters and centimeters respectively.

Do we really need to worry about the elevation in this problem? Why?
Did your value for C surprise you?