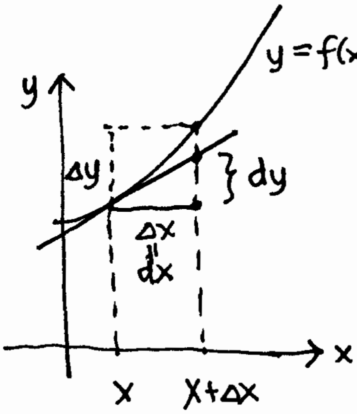
Calc 1 versus Calc 2 : derivative vs integral (notation & associated pictures)

Consider 3 variables x, $y = f(x)$, $z = f'(x) = F(x) = \frac{dy}{dx}$ (like time t, position s, velocity $v = ds/dt$)



$$\underbrace{\frac{dy}{dx}}_{} = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

rate of change of y wrt x
= derivative of y wrt x

as though
$d = \lim \Delta$

limiting ratio of changes

$\boxed{\text{"rate of change" (derivative)} \sim \text{quotient } \div}$

notation: "take derivative" $\frac{d}{dx}$ "wrt x" $\underbrace{(\text{expression})}$ of the expression to immediate right

multiply thru

$$dy = f'(x)\, dx$$

$\boxed{\text{(limiting) change in } y \text{ is a product } *}$

inverse arithmetic operations

differentiation / integration ("inverse operations")

$S \xrightarrow{\text{limit}} \int$

"sum" limit sum like $d = \lim \Delta$

$$\int dy = \int \underbrace{f'(x)}_{F(x)} dx = \int \overbrace{F(x)\, dx}^{\text{"integral of F wrt x"}}$$

sum up small successive changes in y to get total change

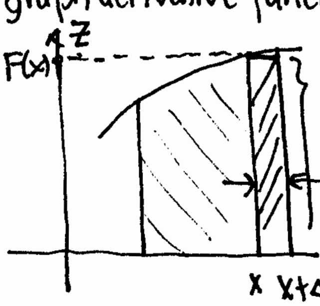
set $z = F(x)$ to graph
yet distinguish from $y = f(x)$

opening/closing delimiters

notation : $\int$ take integral $F(x)$ of this expression dx wrt x

$[\int a^b da \neq \int a^b db]$

graph derivative function:



$\Delta A = F(x)\, dx$ change in area under graph

$\Delta x = dx$

A = area under graph $= \int F(x) dx$

fish hook goes in easy (diff)
but comes out hard (int).

inverse operations often much
harder procedures.