Calculus: Basic Functions for instant recall

	diff	int	simple u-sub
power	$\frac{d}{dx} x^n = n x^{n-1}$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$	$\int (ax+b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$
ln	$\frac{d}{dx} \ln\lvert x\rvert = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \ln\lvert x\rvert + C$	$\int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln\lvert ax+b\rvert + C$
exp	$\frac{d}{dx} e^x = e^x$	$\int e^x \, dx = e^x + C$	$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$
trig	$\frac{d}{dx} \sin x = \cos x$	$\int \cos x \, dx = \sin x + C$	$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$
	$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x \, dx = -\cos x + C$	$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$

chain/ u-sub $\quad \frac{d}{dx} f(u) = \underbrace{f'(u)}_{\frac{df}{du}} \frac{du}{dx}$

$$\int f(u(x)) \frac{du(x)}{dx} \, dx = \int f(u) \, du = F(u) + C = F(u(x)) + C$$

if $F'(x) = f(x)$ "antiderivative"

additive constant $\quad \frac{d}{dx}(f(x) + c) = \frac{d}{dx} f(x)$

multiplicative constant $\quad \frac{d}{dx}(c f(x)) = c \frac{d}{dx} f(x) \qquad \int c f(x) \, dx = c \int f(x) \, dx$

$+ \neq *$!

You are expected to be able to do any of the above explicit integrals by hand, or any that can be reduced to them by an obvious u-substitution. Any derivative or integral you are uncertain of you are expected to check symbolically with Maple or your graphing calculator. There is no excuse for getting a derivative or integral wrong with technology at your fingertips.

MAT 2500 = CALC 3 and MAT 2705 = DEwLinAlg assume these basic operations from CALC 1 and CALC 2 and build on them.

Some Algebra Rules

distributive rule: $a(b+c) \underset{\text{factor}}{\overset{\text{expand}}{=}} ab+ac$ $\quad [a+ac = a\cdot 1 + ac = a(1+c)]$

fractions:

$\frac{a}{b}+\frac{c}{d} = \frac{ad}{bd}+\frac{bc}{bd} = \frac{ad+bc}{bd}$ add numerators if same denominator

$\frac{a}{b}+c = \frac{a}{b}+\frac{bc}{b} = \frac{a+bc}{b}$ (cannot cancel b's in last expression)

$a^{-1}=\frac{1}{a}$, $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$

$\frac{c}{\left(\frac{a}{b}\right)} = \left(\frac{b}{a}\right)c = \frac{bc}{a}$ $\qquad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

[always factor before cancelling: $\frac{a+ac}{ab} = \frac{\cancel{a}(1+c)}{\cancel{a}b} = \frac{1+c}{b}$]

cancellation:

$*$: $\frac{a\cancel{b}}{\cancel{b}} = a$

$+$: $a+\cancel{b-b} = a$

exponentials $(a>0)$:

product	$a^x a^y = a^{x+y}$	$\ln xy = \ln x + \ln y$
quotient	$a^x/a^y = a^{x-y}$	$\ln x/y = \ln x - \ln y$
power	$(a^x)^y = a^{xy}$	$\ln x^p = p \ln x$
reciprocal	$(a^x)^{-1} = a^{-x} = 1/a^x$	$\ln \frac{1}{x} = \ln x^{-1} = -\ln x$

logs $(\ln = \log_e)$:

only base e needed:

$a = e^{\ln a}$

$a^x = (e^{\ln a})^x = e^{(\ln a)x}$

$\log_a x = \frac{\ln x}{\ln a}$

exp/ln properties: $e^{\ln x} = x \ (x>0)$ $\quad e^0 = 1$ $\quad e^1 = e \approx 2.718$

$\ln e^x = x$ $\quad \ln 1 = 0$

$[\, e^{p\ln x} = (e^{\ln x})^p = x^p \,]$ $\qquad \frac{1}{e^x} = e^{-x}$ (preferred)

powers / roots:

product	$(xy)^p = x^p y^p$	$x^0 = 1 \quad (x \neq 0)$
quotient	$(x/y)^p = x^p/y^p$	$1^x = 1$
power	$(x^p)^q = x^{pq}$	$\sqrt{x^2} = \lvert x\rvert = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$
reciprocal	$x^{-p} = 1/x^p$	

$x^{\frac{1}{n}} = \sqrt[n]{x}$, n integer >0 $\quad$ *always convert radical notation to power notation to work with in calculus*

$x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$

$= (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$

$(a+b)^2 \underset{\text{factor}}{\overset{\text{expand}}{=}} a^2+2ab+b^2$ $\quad$ (binomial theorem for higher integer powers)

Some Algebra Rules 2

solving equations

a) linear: $ax+b=0 \rightarrow x=-b/a \quad (a\neq 0)$

b) quadratic: $ax^2+bx+c=0 \rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad (a \neq 0)$ $\quad$ memorize!

related technique: completing the square

$ax^2+bx+c = a(\underbrace{x^2+\tfrac{b}{a}x}) + c$

$(x+\frac{b}{2a})^2 - (\frac{b}{2a})^2$ $\quad$ [since $= x^2 + \cancel{2}x(\frac{b}{\cancel{2}a}) + \cancel{(\frac{b}{2a})^2 - (\frac{b}{2a})^2}$]

$= a(x+\frac{b}{2a})^2 - a\left(\frac{b^2}{4a^2}\right) + c$

$= a(x+\frac{b}{2a})^2 + (c - \frac{b^2}{4a})$ $\qquad$ *dont memorize formula remember technique*

Distinguish between "solving an equation" for an unknown variable which appears in it, as opposed to "simplifying or rewriting" an expression, as in:

$x^2-2x = x(x-2)$ (factored)

Note: $f(x)g(x) = 0 \longrightarrow$ either $f(x)=0$ or $g(x)=0$

$\frac{f(x)}{g(x)} = 0 \longrightarrow f(x)=0$, provided that $g(x)\neq 0$ at a solution of $f(x)=0$

rules of algebra NOT!

$\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$, $\quad \sqrt{a^2+b^2} \neq a+b$

$\frac{c}{a}+\frac{c}{b} \neq \frac{c}{a+b}$ $\qquad \frac{ab+c}{ad} \neq \frac{b+c}{d}$

$\sqrt{x^2} \neq x$ unless x is known to be positive ($\lvert x\rvert$!)

$f(2x) \neq 2f(x)$ (unless f is very special!)

$\sin 2x \neq 2\sin x$

cancellation NOT!

$*$: $\frac{a+\cancel{b}c}{\cancel{b}} \neq a+c$

$+$: $\cancel{b}+a(c-\cancel{b}) \neq ac$

Miscellaneous

$0x=0$ (says nothing about x)

$\frac{0}{x}=0$ when $x\neq 0$

$\frac{x}{0}$ never defined

math is case sensitive: $T \neq t$, $R \neq r$

even roots of negative #'s are complex, not real:

$\sqrt[2n]{x}$, $x<0$ is undefined over real #'s for $n>0$ integer