

$$r = 8(1 + \sin \theta)$$

$$y = 4 \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} r = \frac{4}{\sin \theta} = \frac{4 \csc \theta}{\sin \theta} \\ \\ \\ \end{array}$$

intersection pts:

$$\sin \theta = 0 \\ \theta = 0, 2\pi \\ \text{obvious}$$

$$\frac{\sin \theta}{4} [8(1 + \sin \theta)] = \frac{4}{\sin \theta}$$

$$2 \sin \theta (1 + \sin \theta) = 1$$

$$2 \sin^2 \theta + 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{-2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm 2\sqrt{3}}{2(2)} = \frac{\pm \sqrt{3} - 1}{2}$$

$$\sin \theta = \frac{\sqrt{3} - 1}{2} > 0$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \frac{1}{4}(\sqrt{3} - 1)^2$$

$$= 1 - \frac{1}{4}(3 - 2\sqrt{3} + 1)$$

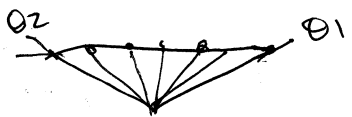
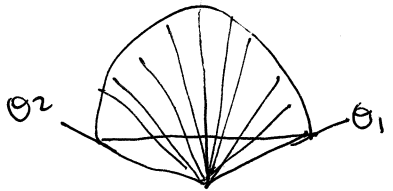
$$= 1 - \frac{1}{4}(4 - 2\sqrt{3})$$

$$= 1 - \frac{1}{2}(2 - \sqrt{3}) = 1 - 1 + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \sqrt{\frac{\sqrt{3}}{2}}$$

$$\theta = \arcsin \left(\frac{\sqrt{3} - 1}{2} \right) = \theta_1 \\ \text{or} \\ \theta = \arccos \sqrt{\frac{\sqrt{3}}{2}}$$

$$\theta_2 = \pi - \theta_1 \\ \cos \theta_2 = -\cos \theta_1 \\ \sin \theta_2 = \sin \theta_1$$



$$A_1 = \int_{\theta_1}^{\theta_2} \frac{r^2}{2} d\theta$$

$$= \int_{\theta_1}^{\theta_2} \frac{1}{2} 8^2 (1 + \sin \theta)^2 d\theta$$

$$= 32 \int_{\theta_1}^{\theta_2} (1 + 2\sin \theta + \sin^2 \theta) d\theta$$

$$32 \left(\theta - 2\cos \theta + \frac{1}{2}(\theta - \frac{1}{2}\sin 2\theta) + \frac{1}{2}(\theta - \sin \theta \cos \theta) \right)$$

$$A_2 = \int_{\theta_1}^{\theta_2} \frac{r^2}{2} d\theta$$

$$= \int_{\theta_1}^{\theta_2} \frac{1}{2} 4^2 \csc^2 \theta d\theta$$

$$= 8 \int_{\theta_1}^{\theta_2} \csc^2 \theta d\theta$$

$$= -\cot \theta \Big|_{\theta_1}^{\theta_2}$$

$$= -\frac{\cos \theta}{\sin \theta}$$

$$A_1 - A_2 = 32 \left(\frac{3}{2}\theta - 2\cos \theta - \frac{1}{2}\cos \theta \sin \theta \right) + 8 \frac{\cos \theta}{\sin \theta} \Big|_{\theta_1}^{\pi - \theta_1} \leftarrow \text{add in } \cos \theta$$

$$= 32 \left[\frac{3}{2}(\pi - 2\theta_1) + 4\cos \theta_1 + \cos \theta_1 \sin \theta_1 \right] - 16 \frac{\cos \theta_1}{\sin \theta_1}$$

$$= 48\pi - 96\theta_1 + 128\cos \theta_1 + 32\cos \theta_1 \sin \theta_1 - 16 \frac{\cos \theta_1}{\sin \theta_1} \approx 204.1600084 \approx \boxed{204.16 \text{ m}^2}$$