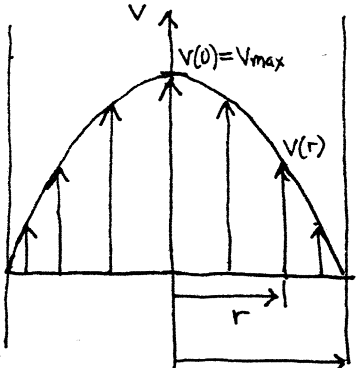
BLOODFLOW MODEL UNDERSTOOD USING VELOCITY PARAMETERS



velocity profile

velocity profile model:

$$v(r) = \underbrace{\frac{P}{4\eta l}}_{\text{coefficient } C = \frac{P}{4\eta l}}(R^2 - r^2) = C(R^2 - r^2) = \frac{v_0}{R^2}(R^2 - r^2) = v_0\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$v_0 \equiv v(0) = CR^2 \rightarrow C = \frac{v_0}{R^2}$$

$$\boxed{\text{where } v_0 = \frac{PR^2}{4\eta l}}$$

most useful representation of functional relationship:

$$\boxed{v(r) = v_0\left(1 - \left(\frac{r}{R}\right)^2\right)} \leftrightarrow \underbrace{\frac{v(r)}{v_0}}_{y} = 1 - \Big(\underbrace{\frac{r}{R}}_{x}\Big)^2$$

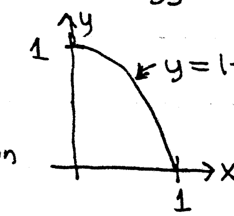
dimensionless relationship

dimensionless variables

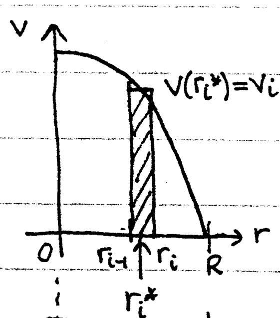
x measures the radius in multiples of R

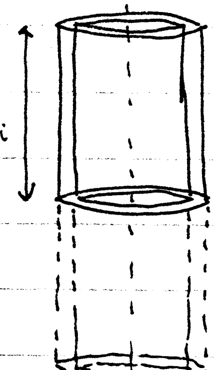
y measures the velocity in multiples of v_0

Setting $C = 1 = R$ or equivalently $v_0 = 1 = R$ in order to plot this relationship with technology is equivalent to switching to the dimensionless variables x and y:



To analyze the total blood volume flowing per time unit through a circular plane cross-section, we use a Riemann integral piecewise constant approximation to develop an integral formula.





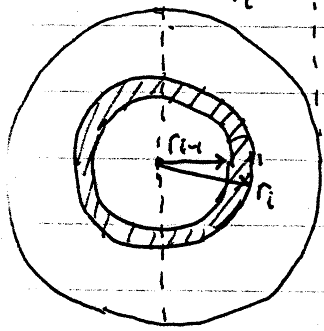
$$\Delta d_i = v_i \underbrace{\Delta t}_{1} = v_i$$

moves here during one time unit.

was here in 1 time unit preceding

In one time unit this volume passes through the ring strip of the circular cross-section

$$\underbrace{\Delta V_i}_{\text{volume of blood}} \approx v_i \Delta A_i = (2\pi r_i^*) v_i \Delta r$$



circular cross-section perpendicular to flow

$$\Delta A_i \approx 2\pi r_i^* \Delta r$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta A_i = \int_0^R 2\pi r\, dr = \pi r^2\Big|_0^R = \pi R^2$$

$$[dA = d(\pi r^2) = 2\pi r\, dr]$$

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta V_i = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi r_i^* v(r_i^*)\Delta r$$

units: velocity × area = volume / time

$$= \int_0^R 2\pi r\, v(r)\, dr$$

$$= \int_0^R 2\pi v_0\left(r - \frac{r^3}{R^2}\right)dr = 2\pi v_0\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\Big|_0^R$$

$$= \underbrace{(\pi R^2)}_{A}\underbrace{\left(\frac{v_0}{2}\right)}_{\bar{v}}$$

Why does it make sense to consider $\bar{v} = \frac{v_0}{2}$ to be the average velocity?

Consider

$$\frac{V}{A} = \frac{\int_0^R v(r)\cdot 2\pi r\, dr}{\int_0^R 2\pi r\, dr} = \frac{\int_{r=0}^{r=R} v(r)\, dA}{\underbrace{\int_{r=0}^{r=R} dA}_{A}} = \lim_{n\to\infty} \sum_{i=1}^{n} v(r_i^*)\underbrace{\left(\frac{\Delta A_i}{A}\right)}_{\text{fractional area}} = \bar{v}$$

fractional area with given velocity: proportional to number of blood particles with this velocity.

This is a weighted average of the velocity, weighted by fractional area per velocity interval.

Thus the volume of blood flowing per unit time is equivalent to a uniform velocity profile with the average velocity: area × avg velocity