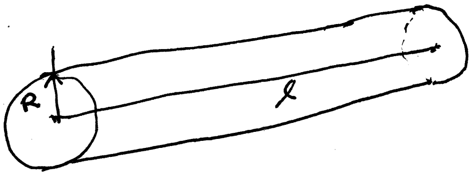
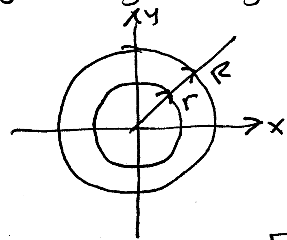
Stewart 6.5.18 Average blood flow velocity.

radially symmetric distribution of fluid flow along a long cylindrical tube.





$0 \leq r \leq R$

$$V = \frac{P}{4\eta\ell}(R^2 - r^2) = \underbrace{\frac{PR^2}{4\eta\ell}}_{V_{max}}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$\boxed{\text{dimensionless variables}}$

define: $x = r/R$

$y = V/V_{max}$

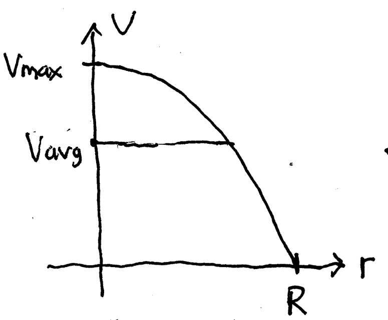
get dimensionfree relationship

$y = 1 - x^2$

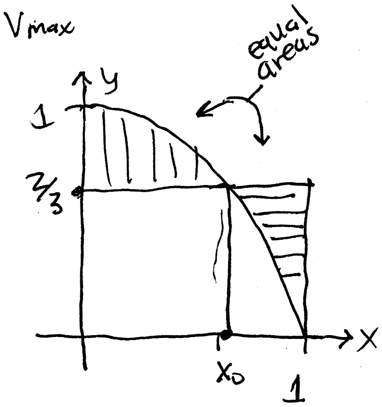
equivalent to setting

$R = 1 = V_{max}$

by measuring distance in multiples of R and velocities in multiples of V_{max}.



$\longleftrightarrow$



"blood" moves fast in center, slow at edge of vessel.

<u>average value:</u>

$$V_{avg} = \frac{1}{R}\int_0^R V(r)\,dr$$

$$= \frac{1}{R}\int_0^R \frac{PR^2}{4\eta\ell}\left(1 - \frac{r^2}{R^2}\right)dr$$

$$= \frac{PR}{4\eta\ell}\underbrace{\left(r - \frac{r^3}{3R^2}\right)\Big|_0^R}_{\frac{2}{3}R} = \frac{2}{3}\left(\frac{PR^2}{4\eta\ell}\right)$$

$$= \frac{2}{3}V_{max}$$

$$y_{avg} = \frac{1}{1}\int_0^1 1 - x^2\,dx = x - \frac{x^3}{3}\Big|_0^1 = \frac{2}{3}$$

where cross? $1 - x^2 = 2/3$ $(x \geq 0)$

$x^2 = 1/3$

$x = 1/\sqrt{3} \approx 0.57735$

$\equiv x_0$

$\boxed{\text{dimensionless variables can often simplify a problem!}}$

If radial variation of velocity not so important, can replace model of blood flow by uniform velocity of V_{avg}.