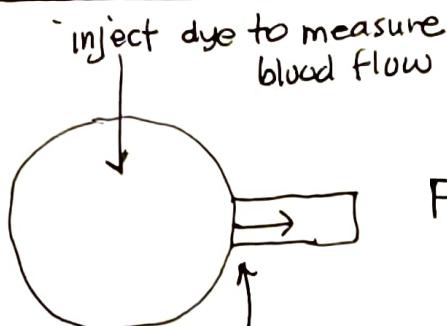


8.4b

Blood Flow APPS

①

cardiac output

How to measure how much blood is being pumped out of the heart?

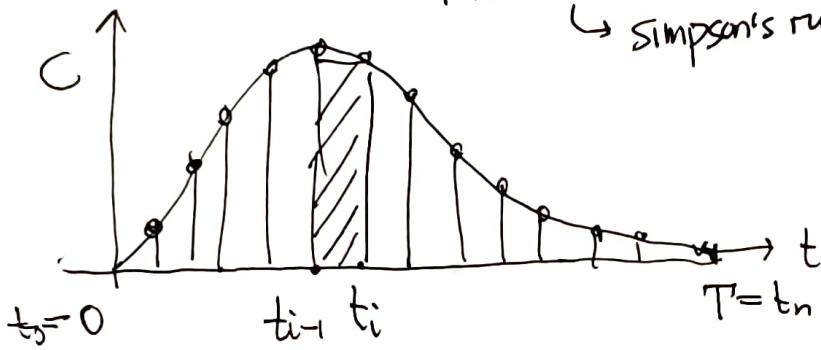
$$F = \text{blood flow rate} \quad (\text{unknown}) \quad \left( \frac{\text{Vol}}{\text{time}} \sim \frac{L}{\text{sec}} \right)$$

probe sensor  
measures concentration  $c(t)$   $\left( \frac{\text{mass}}{\text{Vol}} \sim \frac{\text{mg}}{L} \right)$   
of dye in blood at time  $t$   $(\text{time} \sim \text{sec})$

until dye is no longer detectable at time  $T$ :  
 $0 \leq t \leq T \rightarrow t \Rightarrow t_i, i=0, \dots, n$  (even)

$$\Delta t = \frac{T}{n}$$

tabular data (see Maple)  
↓ Simpson's rule!



$$\sum_{i=1}^n Fc(t_i^*) \Delta t \rightarrow \int_0^T Fc(t) dt = A$$

amount of dye  
passing thru  
sensor per sec

$$\left( \frac{L}{\text{sec}} \right) \left( \frac{\text{mg}}{L} \right) \sim \frac{\text{mg}}{\text{sec}}$$

dye passing per sec

$$F = \frac{A}{\int_0^T c(t) dt} \quad \left( \frac{L}{\text{sec}} \right)$$

total amount of dye given:  $A(\text{mg})$

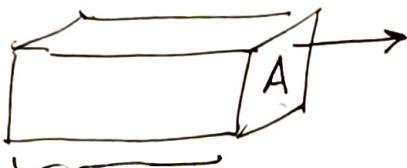
8.4b

Blood Flow Apps

(2)

Blood Flow in Arteries

Suppose a liquid is flowing at uniform speed  $v$  across a cross-sectional area  $A$ .



$$L = v \Delta t = \text{distance traveled in time } \Delta t$$

Box volume passing area  $A$  in time  $\Delta t$ :

$$V = L A = v A \Delta t$$

The flow rate is volume/time:

$$F = \frac{V}{\Delta t} = \frac{v A \Delta t}{\Delta t} = v A$$

Blood flows in long cylinders (radius  $R$ )



length:  $l$

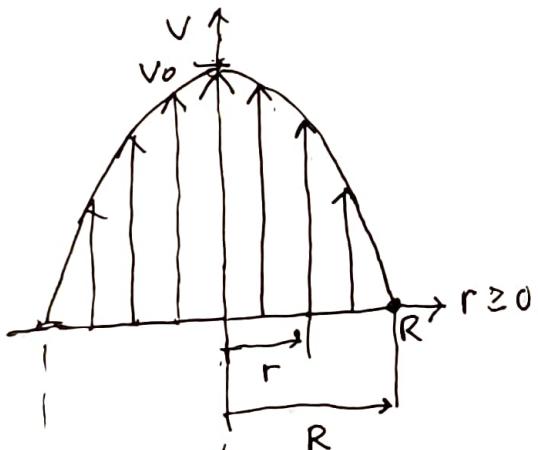
faster in center,

slower near walls

(friction slows blood down)

so the velocity depends on the radius from the center.

The simplest "velocity profile" to describe this is quadratic,



$$0 \leq r \leq R :$$

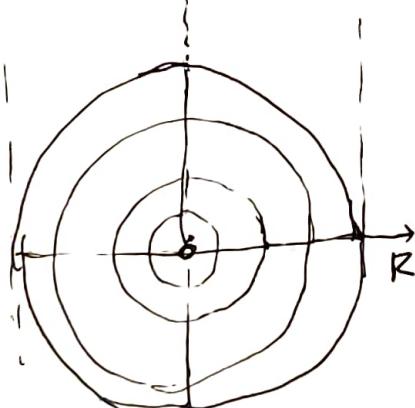
$$v(r) = v_0 \left(1 - \left(\frac{r}{R}\right)^2\right) \quad \text{or} \quad \frac{v(r)}{v_0} = 1 - \left(\frac{r}{R}\right)^2$$

max velocity model:

$$U = 1 - x^2 \quad \begin{cases} \text{(dimensionless variables to plot)} \end{cases}$$

$$v_0 = \frac{P R^2}{4 \eta l} \quad \begin{cases} \text{bigger radius, easier to flow} \\ \text{longer means slower to reach} \\ \text{viscosity (bigger slows down flow)} \end{cases}$$

blood pressure, more means more flow

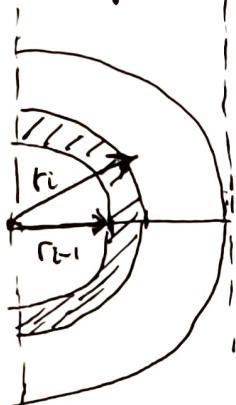
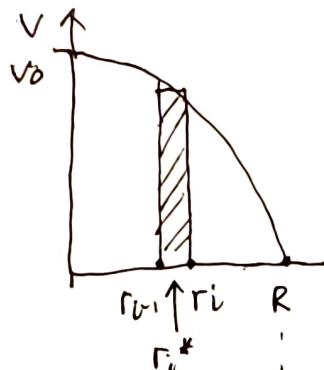


We need the Riemann approach to develop an integral formula treating each thin layer as approximately a uniform flow at fixed velocity.

8.4b

Blood Flow Apps

(3)

BLOOD Flow in Arteries

circular cross-section  
perpendicular to flow

$$\Delta A_i \approx (2\pi r_i^*) \Delta r$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \Delta A$$

$$= \int_0^R 2\pi r dr$$

$$= \pi r^2 \Big|_0^R$$

$$= \pi R^2$$

$$\int dA = 2\pi r dr$$

$$\int dA = A$$

$$F_i = V_i (2\pi r_i^*) \Delta r$$

amount per unit time  
crossing a plane cross-section

$$F = \lim_{n \rightarrow \infty} \sum_{i=1}^n F_i$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n V(r_i^*) 2\pi r_i^* \Delta r$$

$$= \int_0^R V(r) \cdot 2\pi r dr$$

$dA$  differential  
of area

↓ use quadratic  
model

$$F = \int_0^R V_0 \left(1 - \left(\frac{r}{R}\right)^2\right) \cdot 2\pi r dr$$

$$= 2\pi V_0 \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

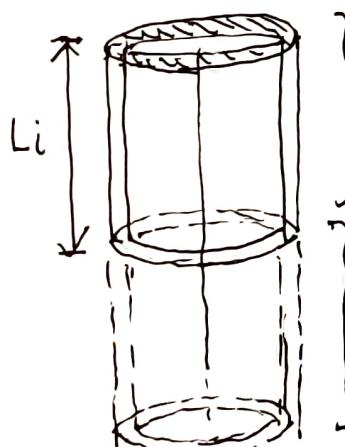
$$= 2\pi V_0 \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) \Big|_0^R$$

$$= 2\pi V_0 \left(\frac{R^2}{2} - \frac{R^2}{4}\right)$$

$$= \frac{\pi R^2 V_0}{2}$$

$$= (\pi R^2) \left(\frac{V_0}{2}\right) = A \bar{V}$$

↑  
equivalent  
uniform velocity!



moves here during  
one time unit  
distance  $L_i = V_i \cdot dt$

↑  
shell was here  
in one time unit  $\Delta t$   
prior

ring of area  $\Delta A_i$  moves past horizontal plane  
of volume  $L_i \Delta A_i = \underbrace{V_i \Delta A_i}_{\text{volume flow}} \Delta t$   
per unit time.

8.1b

Blood Flow Apps

④

Blood Flow in Arteries

$\bar{V}$  is the area averaged velocity:

$$\bar{V} = \frac{\int E}{A} = \frac{\int_0^R v(r) dA(r)}{\int_0^R dA(r)} = \frac{\int_0^R v(r) 2\pi r dr}{\int_0^R 2\pi r dr}$$

this is a weighted average that weights each velocity by the circumference of the ring at radius  $r$

The velocity contributes corresponding to the fractional area each ring represents.

Outer rings have more cross-sectional area and must contribute more to the integration process.

One can simply replace the complicated blood flow by the average velocity to understand flow per unit time without worrying about the details.

Plug back in parameters:

$$F = \frac{\pi R^2}{2} \left( \frac{PR^2}{4\eta l} \right) = \frac{\pi P R^4}{8\eta l}$$

very sensitive to radius of arteries.

cardiovascular disease:  
plaque builds up on walls  
& restricts flow by lowering  
radius  $R$

Blood pressure is the key marker for problems.

Suppose we compare pressures at the original radius  $R_0$  and

a later smaller radius  $R < R_0$ :

If we are to maintain the same flow:  $PR^4 = P_0 R_0^4$  or

$$\frac{P}{P_0} = \left( \frac{R_0}{R} \right)^4$$

So decrease radius by 25%, the pressure increases by  $\left(\frac{P}{P_0}\right) = \left(\frac{R_0}{0.75R_0}\right)^4 \approx 3.16$ !

more than triples the blood pressure  
(Stewart B.4.20)