

Business majors now take the sci/eng calc sequence if they are interested in finance, which is math intensive.

[The first "quants" were math-physics guys (yes, male) because business people lacked the math skills.]

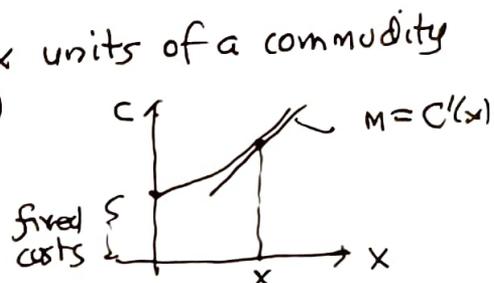
MAT150 = CALC I briefly introduces business jargon in Stewart Calc 3.7 which those who skip 1500 with AP credit miss, so here is a summary. Section 4.7 completes the intro.

BUSINESS JARGON

COST FUNCTION

$C(x)$ = cost of producing x units of a commodity (or service rendered)

increasing function!



MARGINAL COST

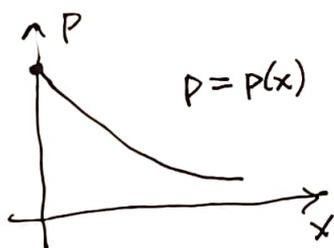
$C'(x) \approx \frac{C(x+1) - C(x)}{1}$ = cost of producing one additional unit for $x \gg 1$!

DEMAND FUNCTION

$p(x)$ = price per unit company can charge if x units are sold, or # units that will be sold at a given price

decreasing function!

lower price, more demand — more units sold



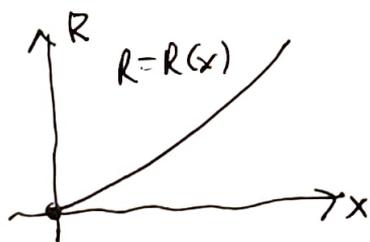
REVENUE FUNCTION

$R(x)$ = (quantity sold) (price per unit)
= $x p(x)$

increasing function!

sell more, take in more revenue!

sell none, take in none



MARGINAL REVENUE

$R'(x) \approx \frac{R(x+1) - R(x)}{1}$ = additional revenue from selling 1 more unit for $x \gg 1$!

8.4a

Business Calc minilesson

②

PROFIT FUNCTION

$$P(x) = R(x) - C(x) = \text{revenue minus cost}$$

(in) (out)

MARGINAL PROFIT

$$P'(x) = R'(x) - C'(x)$$

$= 0$ when profit maximized:
marginal revenue = marginal cost

Example 4.7.4

Store is selling 200 TVs/week at \$350 each. [x=200, p=350]

Market survey indicates for each \$10 rebate offered buyers,
the number of TVs sold will increase by 20/week [Δx=20]

Find the demand & revenue functions.

How large a rebate should be offered to maximize revenue?

$$x - 200 = \text{excess sales}$$

$$\frac{x - 200}{20} = \text{multiples of 20 excess sales} \longleftrightarrow \text{each } \$10 \text{ in price decrease}$$

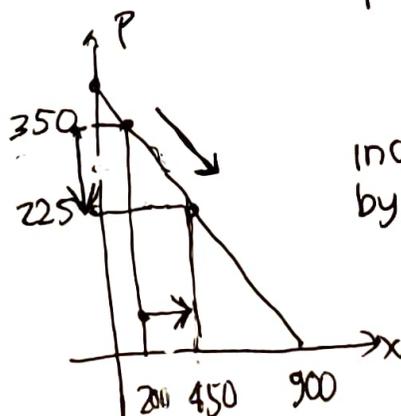
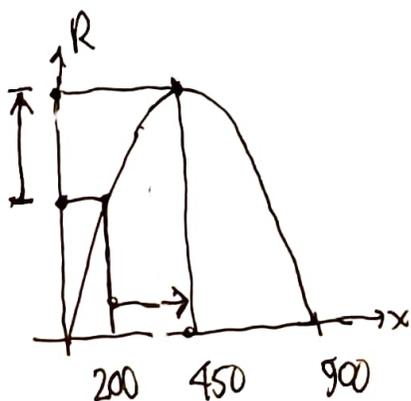
$$p(x) = \underbrace{350}_{\text{current price}} - \underbrace{\left(\frac{x - 200}{20}\right)(10)}_{\text{decrease due to enough rebates to achieve } x \text{ sales}}$$

$$= 350 - \frac{x}{2} + 100 = 450 - \frac{x}{2}$$

$$R(x) = xp(x) = x\left(450 - \frac{x}{2}\right) = 450x - \frac{x^2}{2}$$

$$R'(x) = 450 - x = 0 \rightarrow x = 450$$

$$\rightarrow p = p(450) = 450 - \frac{450}{2} = 225$$



Increase demand
by lowering price \rightarrow
increase revenue

8.4a Business Calc minilesson

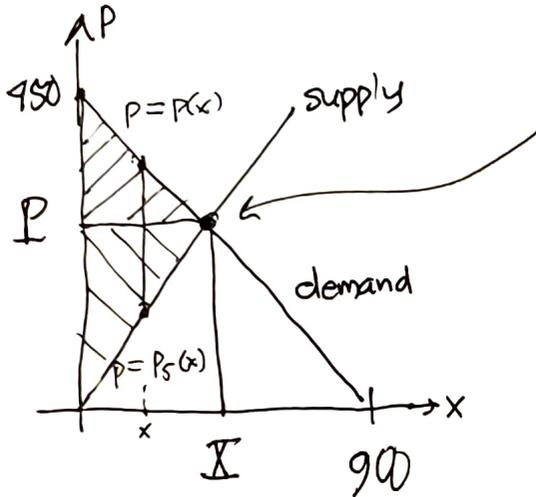
3

But what about supply? We always hear about "supply and demand!"

SUPPLY FUNCTION $p_s(x)$ = price producers willing to accept if they produce x units

increasing function:

producers willing to produce more at higher prices, none at zero price (starts at origin)



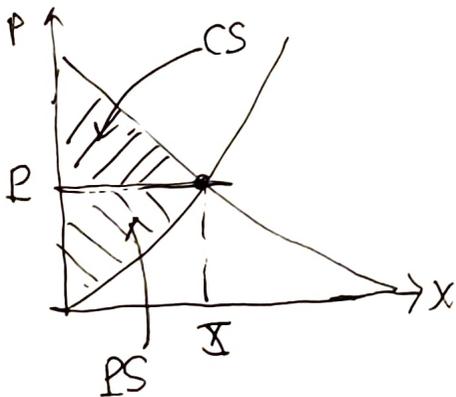
"market equilibrium"

consumers are willing to buy what producers are willing to produce

see Stewart 8.4 exercises for supply function

If we have a linear supply function $p = mx$ then the marginal supply (derivative) is the slope m .

As we change m we move along the demand and revenue curves as the equilibrium point moves, independent of maximizing revenue.



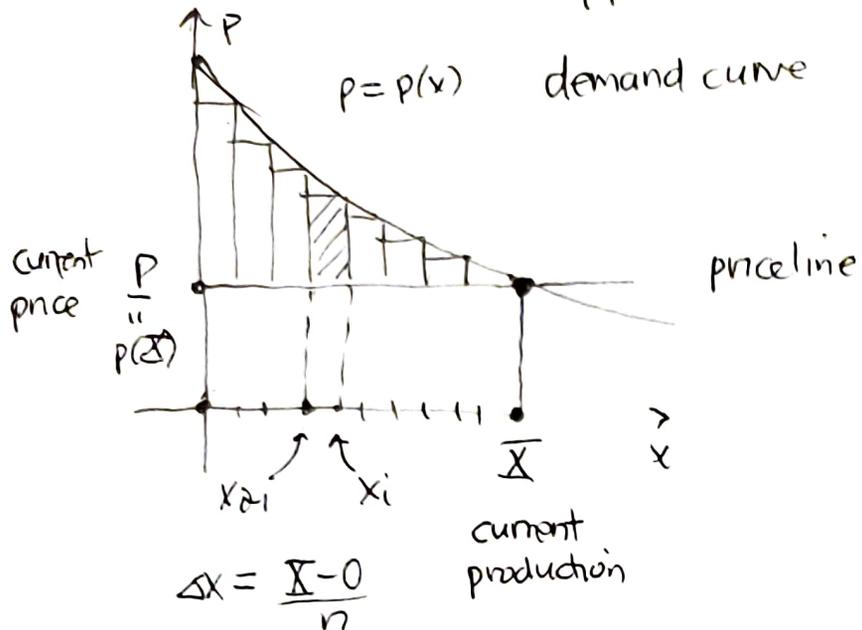
For any pair of supply and demand functions at the equilibrium point,

CONSUMER SURPLUS $= \int_0^X p(x) - P \, dx$
 = area between demand curve and equilibrium price line

PRODUCER SURPLUS $= \int_0^X P - p_s(x) \, dx$
 = area between the supply curve and equilibrium price line.

8.1a Business Calc MiniLesson

Surplus? Why? Consider the consumer surplus. For interpretation we need the Riemann approach.



ith interval, use right endpoint Riemann approximation.

For this range of items produced, consumers would have been willing to pay $p(x_i) - P$ more for the items, spending $(p(x_i) - P) \Delta x$ more than P to purchase these Δx items.

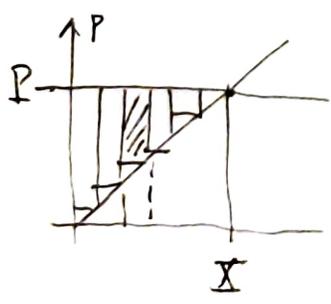
$\sum_{i=1}^n (p(x_i) - P) \Delta x =$ amount extra consumers would have been willing to pay for all the items total (some)

so in some sense it is what the consumers as a group have saved by buying X items at the price P .

$CS(X) = \int_0^X p(x) - P dx =$ "consumer surplus"

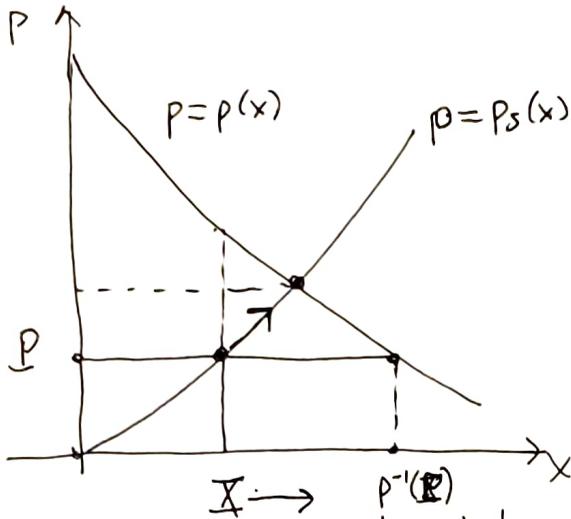
producer surplus is the complementary discussion for the supply curve.

the total amount more the producers have gained for producing the items at price P compared to the lower prices they would have accepted for producing less.



$PS(X) = \int_0^X P - p_s(x) dx$

Market equilibrium forces



undersupply

price and production too low to meet demand

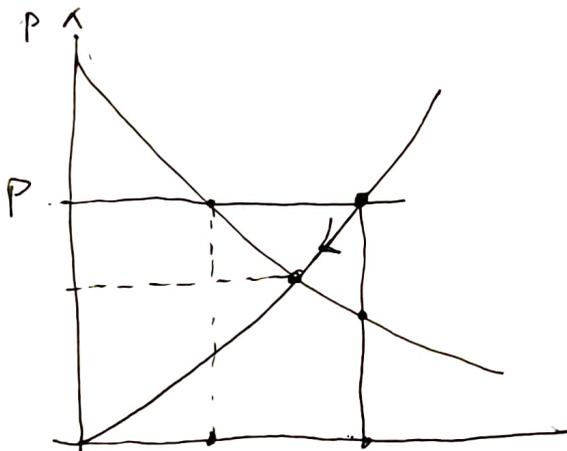
producers increase both as long as consumers willing to buy

demand at this price greater than supply X

(price too low)

raise price

until they meet at equilibrium



oversupply

demand at this price lower than supply X

price and production too high for actual demand - producers decrease price & production to meet demand until consumers will buy all produced.

lower price - (price too high)

If only life were so simple! Free markets do not exist. Much more complicated