

DISCLAIMER] I have no clue what it is like to be a student in today's American high schools, BUT I see the results in entering freshmen.

Mathematical notation] is a precise and efficient shorthand for complicated mathematical ideas and unless you are comfortable with it, you do not understand the underlying ideas.

Repeating a recipe to "solve" a mathematical problem like one you've seen before is meaningless. To effectively use math in a STEM field, you need to understand the underlying ideas so that you can apply them in unfamiliar settings,

No x, y, z, t , no $f(x)$, no named functions, etc. These familiar symbols for "typical" math problems trigger our mental associations with mathematical activities but we need to be ready to transfer those activities to applications WITHOUT these familiar symbols.

MAT1505 assumes you are familiar with function, derivative and integral notation through section 15.3 of Stewart Calculus. Let's review it briefly.

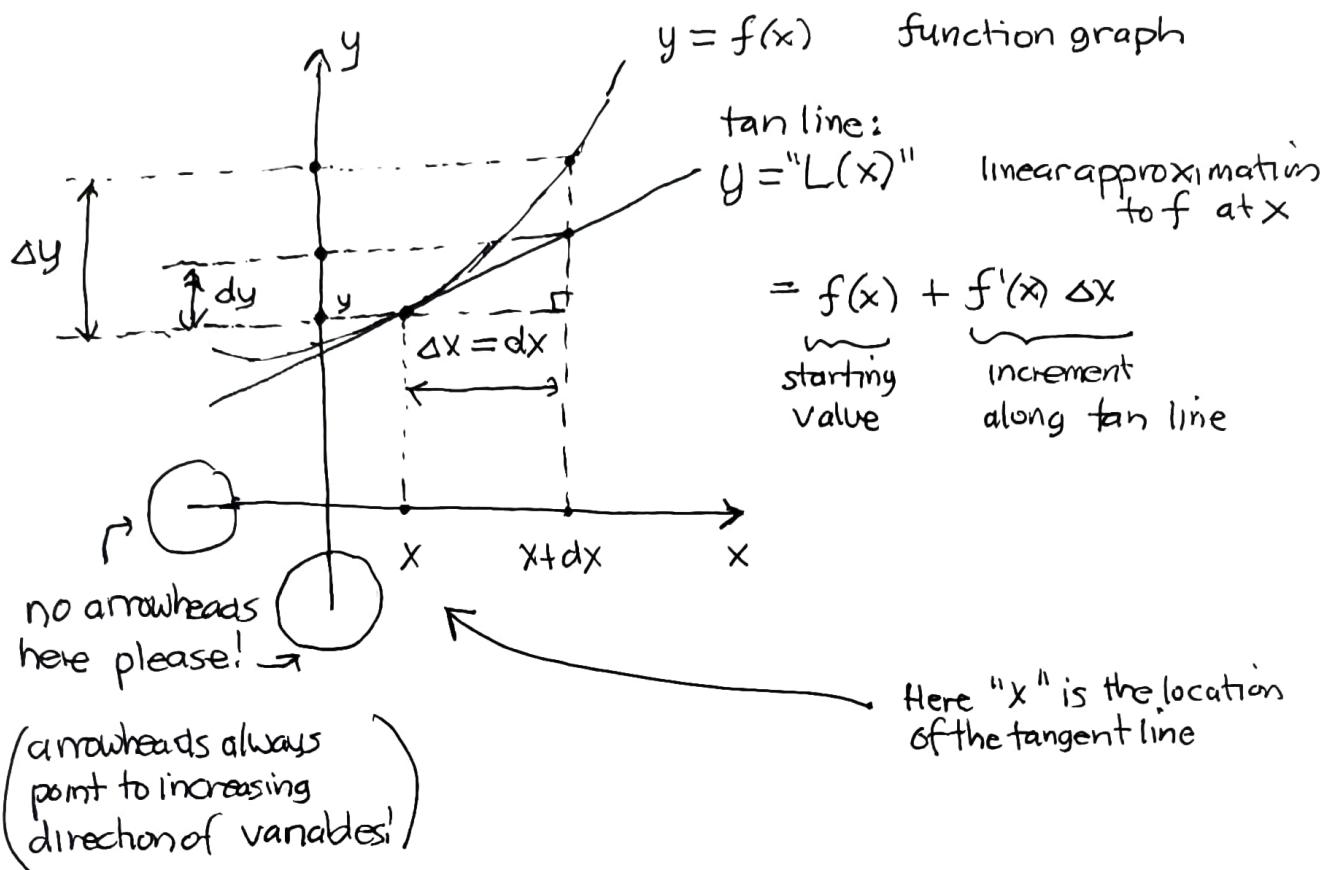
I will post lecture notes like these for each class so that you do not need to take notes but can LISTEN carefully to me as I explain new material and present typical examples. Stop me during my lecture if you do not understand something.

Although you can read these in advance, they lack my oral delivery explaining the printed words. Keep this in mind.

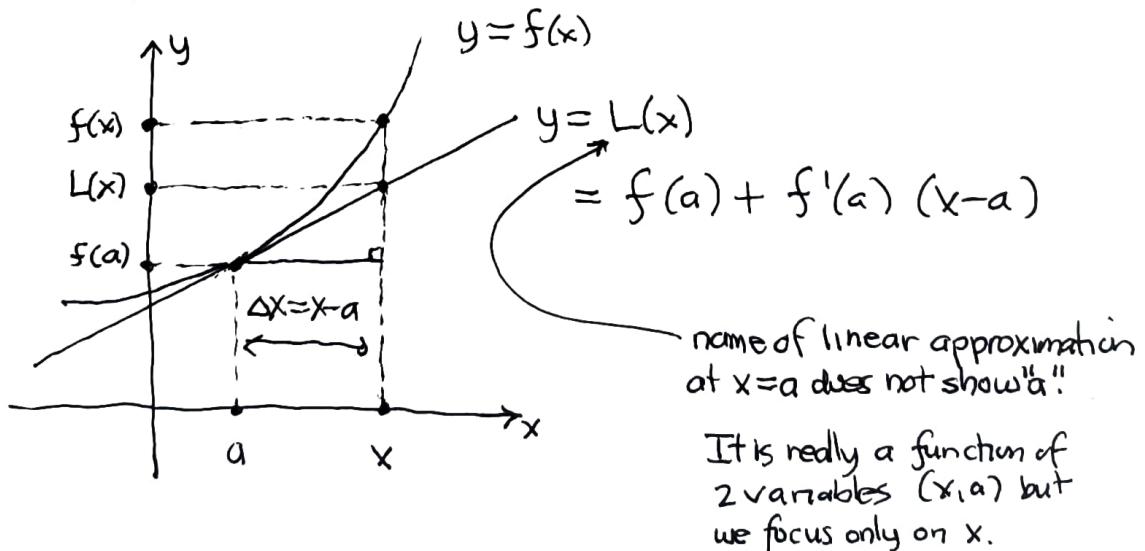
15.3 diff/int notation

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usual derivative illustration:



The quotes $y = "L(x)"$ are necessary because of the limitations of calculus textbook notation for the linear approximation function. This is sidestepped by using a symbol for a particular value of x namely : $x=a$.



15.3 diff/int notation

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derivative notation

the "deedee x" notation makes sense as a quotient

$$y = f(x)$$

$$\frac{dy}{dx} = \underbrace{\frac{d}{dx}}_{\text{take derivative wrt } x \text{ of expression to the right}} (f(x)) = \frac{df(x)}{dx} = \underbrace{f'(x)}_{f' \text{ is the name of the new derivative function}}$$

wrt x of expression to the right

surrounded by parentheses when needed

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \right) \quad \text{so "d" is like "lim Δ"}$$

differential notation

the numerator and denominator make sense individually

$$\frac{dy}{dx} = f'(x) \longrightarrow dy = f'(x) dx \quad \left(= \cancel{\frac{dy}{dx}} \right) \quad \text{differentials cancel!}$$
$$= d f(x)$$

The derivative is defined as the quotient of two "differentials":

$$dx = \Delta x \quad (\text{ind. var.})$$

$$\frac{dy}{dx} = \frac{f'(x) dx}{dx} = f'(x)$$

$$dy = f'(x) dx \quad (\text{dep. var.})$$

The derivative is a [quotient] with quotient units in applications

The differential is a [product] with the same units as the function.

15.3 definite integration and antiderivatives

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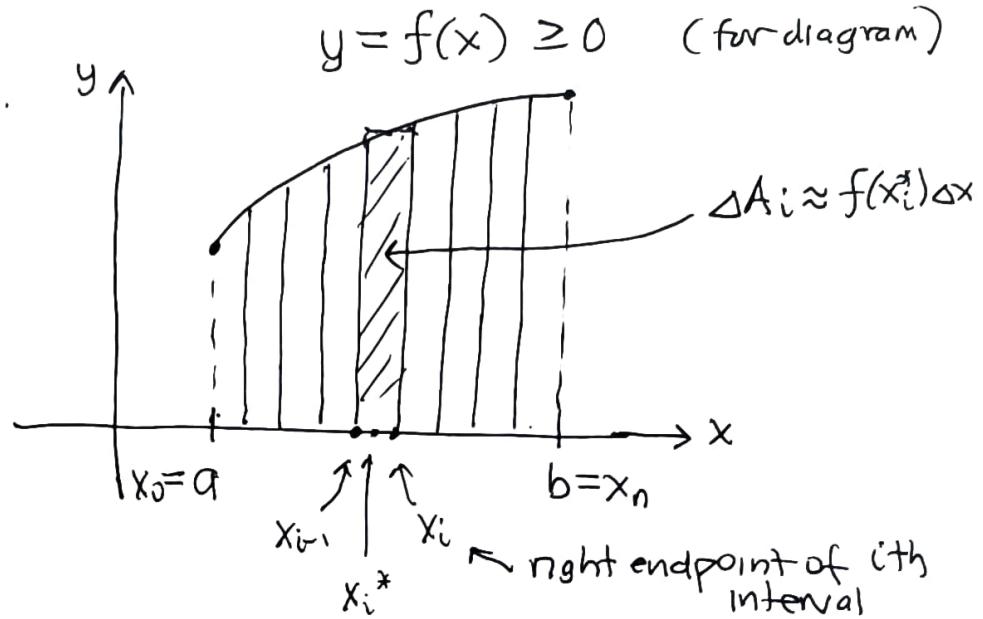
Divide up interval

$$a \leq x \leq b$$

into n equal subintervals of

$$\text{width } \Delta x = \frac{b-a}{n}$$

$$x_i = a + i \Delta x$$



For a nonnegative function

$f(x) \geq 0$ the integral of f wrt x over this interval

is the area between the graph and the horizontal axis.

$$A \approx \sum_{i=1}^n \Delta A_i \approx \sum_{i=1}^n f(x_i^*) \Delta x$$

↓ ↓ ↓
limit as $\Delta x \rightarrow 0$

$\sum \rightarrow S \rightarrow \int$
Greek Latin for "sum" stretch

$$\int_a^b f(x) dx$$

"int ... dee x" notation

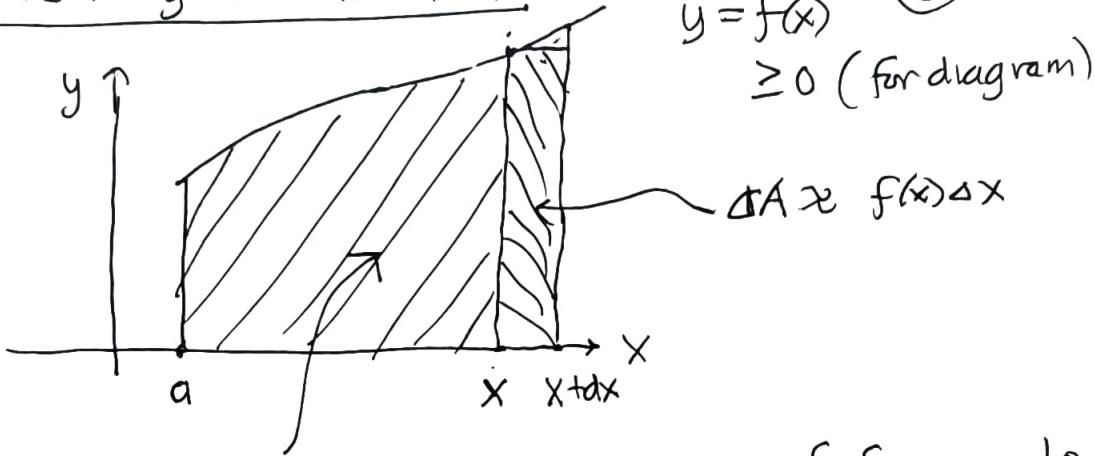
opens and closes integrand expression
like matching "delimiters"

"dx" identifies variable of integration
the "limits" identify the interval endpoints

For general $f(x)$ taking all real values, this integral

is the difference of the area above the axis and the area below the axis: "signed" area.

15.3 definite integral evaluation



$$y = f(x) \quad (5)$$

≥ 0 (for diagram)

Let $A(x) = \int_a^x f(u) du$ = area under f from a to x
 "dummy variable" ("area accumulation function")
 with reference point $x=a$

$$\begin{aligned} \text{Then } A(x+\Delta x) &= \int_a^{x+\Delta x} f(u) du \\ &= \int_a^x f(u) du + \underbrace{\int_x^{x+\Delta x} f(u) du}_{\Delta A \approx f(x) \Delta x} \\ &\quad \downarrow \qquad \downarrow \quad \text{limit as } \Delta x \rightarrow 0 \\ dA &= f(x) dx \end{aligned}$$

$A'(x) = \frac{dA}{dx} = f(x)$ This says A is an
 "antiderivative" of f

$$\frac{d}{dx} A(x) = f(x)$$

$$\boxed{\frac{d}{dx} \left(\int_a^x f(u) du \right) = f(x)} \quad \text{"Fundamental Thm of Calculus"}$$

And

$$A(b) = \int_a^b f(x) dx = A(b) - \underbrace{A(a)}_0 = A(x) \Big|_a^b$$

Let $F(x) = A(x) + C$ be any other antiderivative (C a constant)

$$F'(x) = A'(x) + 0 = A'(x) = f(x)$$

$$F(b) - F(a) = [A(b) + C] - [A(a) + C] = A(b) = F(x) \Big|_a^b$$

so

$$\boxed{\int_a^b f(x) dx = F(x) \Big|_a^b \quad \text{for } F'(x) = f(x)}$$

evaluation mechanism

5.3 differentiation & antiderivation

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$$\begin{array}{ccc} F(x) & \xrightarrow{\text{differentiation}} & f(x) = F'(x) \\ F(x) & \xleftarrow{\text{antiderivation}} & f(x) \end{array}$$

"inverse operations" (almost!)



a fishhook goes in easy (diff)
but comes out hard (int)

inverse operations are often much
harder procedures than the original
operation!

BUT evaluating the derivative or antiderivative of
an explicit expression like $f(x) = e^{3x} \cos 2x$ is the
LEAST important aspect of calculus - they are just
mechanical calculations best done by computers!

Similarly using algebra to solve a particular equation
 $f'(x) = 0$ or $f(x) = 0$ is also a mechanical calculation.

Yes, we need to do all of these operations for simple enough
expressions and equations to understand how the ideas of
calculus work in practice, but it is more important to know
WHEN we need to do the operations. We need to understand
the sequence of steps one must take to carry out calculations,
but real life is often much more complicated than our
simple practice problems.