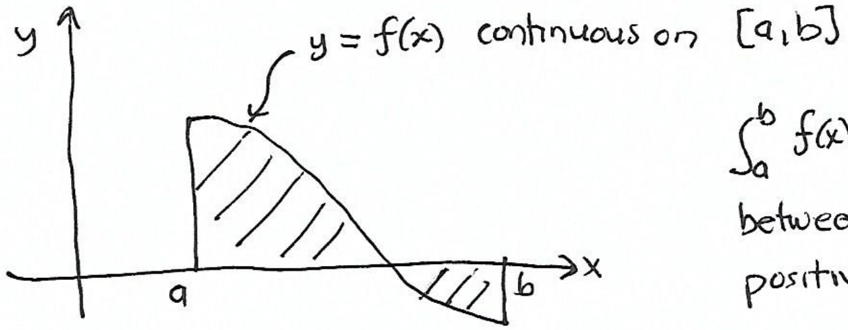
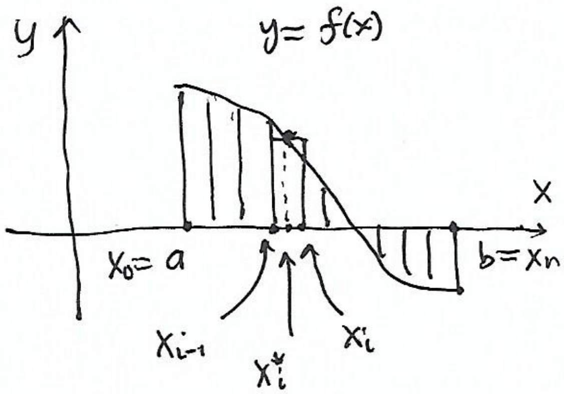
15.0 **Riemann definition of a definite integral**



$\int_a^b f(x)\,dx$ represents the net area between the graph and the x-axis: positive above, negative below.

This interpretation comes from the Riemann definition of the definite integral.



$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i^*)\,\Delta x}_{}$$

signed rectangular area approximating subinterval contribution to integral

$\Delta x = (b-a)/n$ width of subinterval

But this definition is not useful for evaluating the integral.
The Fundamental Theorem of Calculus (FTC) instead uses antiderivatives to evaluate integrals.

$$\int_a^b f(x)\,dx = F(b) - F(a) \equiv F(x)\Big|_a^b$$

where $F'(x) = f(x)$, $F(x)$ is any antiderivative of $f(x)$

Tomorrow we explain why this works.

However, we still need the Riemann definition to derive new integration formulas for quantities more general than signed area enclosed by graphs.

Since $\frac{d}{dx}(F(x) + \underset{\substack{\uparrow \\ \text{constant}}}{C}) = F'(x) + 0 = f(x)$ there is a 1-parameter family of antiderivatives of any function called the $\boxed{\text{indefinite integral}}$.

$$\int f(x)\,dx = F(x) + \underbrace{C}_{\text{"arbitrary constant"}}$$

$\boxed{15.0}$ Riemann definition of a definite integral

We dont need complicated functions to illustrate calculus ideas.

Ex. $f(x) = x^4$

$$\int x^4\,dx = \underbrace{\frac{x^5}{5}}_{} + C \qquad \text{(power rule)}$$

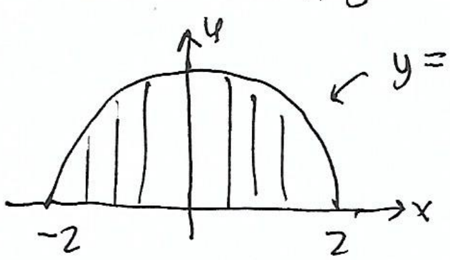
$F(x) = \frac{x^5}{5}$ is the antiderivative for which $F(0) = 0$.

check. $\frac{d}{dx}\left(\frac{x^5}{5} + C\right) = \frac{5x^4}{5} + 0 = x^4$ ✓

When dealing with complicated functions we can use technology to provide the required antiderivatives.

Ex $$\int \sqrt{4-x^2}\,dx \underset{\text{Maple}}{=} \frac{x\sqrt{4-x^2}}{2} + 2\arcsin\left(\frac{x}{2}\right) + C$$

we will never learn how to derive this (last century curriculum!) but we need it for the area of a circle!



$y = \sqrt{4-x^2} \Leftrightarrow x^2+y^2 = 4$
$y \geq 0$

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx = \underbrace{\frac{\pi(2)^2}{2}}_{\text{half area of circle}}$$

Technology empowers us to extend our pencil and paper calculational skills! We need to learn how to use it as a tool.

$\boxed{\text{See Maple now}}$ (bob projects worksheet)

You can use technology to check all your calculations on tests / quizzes but not substitute for hand calculations which must be clearly communicated on paper.