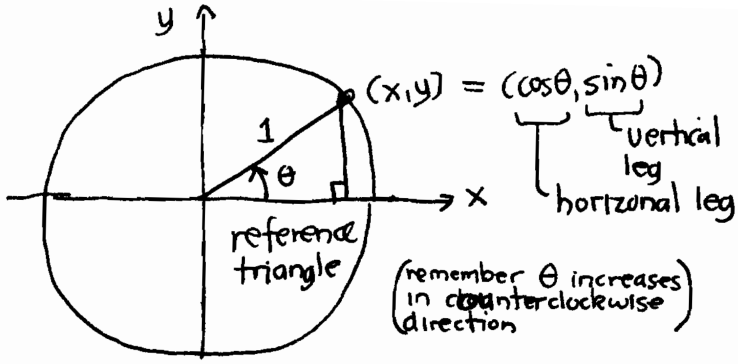
TRIG

unit circle:
$x^2+y^2=1$



$\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

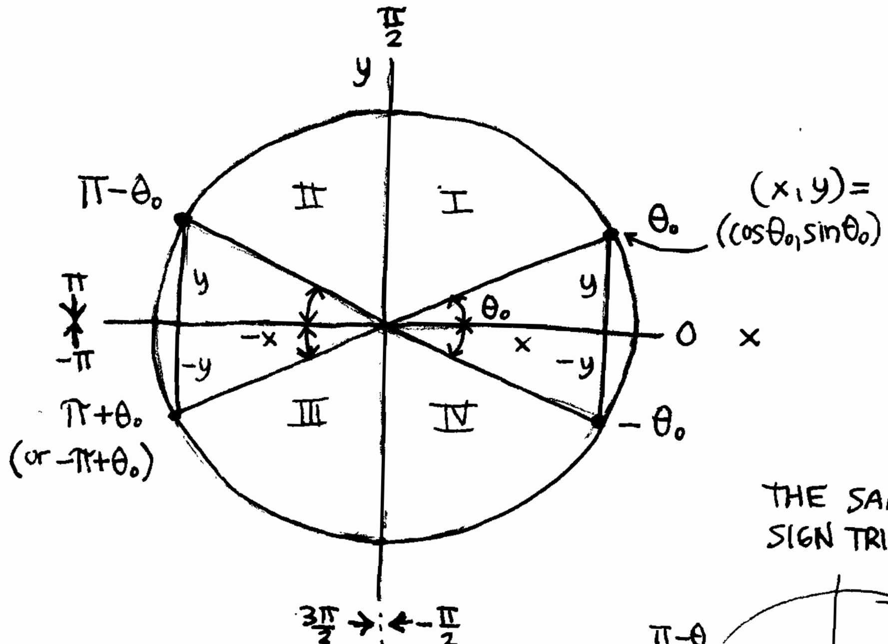
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

divide by $\cos^2\theta$: $1 + \tan^2\theta = \sec^2\theta$
divide by $\sin^2\theta$: $\cot^2\theta + 1 = \csc^2\theta$

For each angle θ we can draw a reference triangle hanging off the x-axis with acute reference angle $\theta_0 \in [0, \frac{\pi}{2}]$

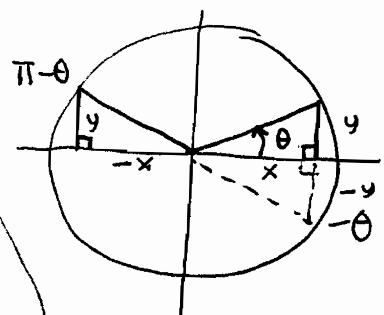


The sign of the trig functions in the II, III, IV quadrants can be computed using the reference triangle.

EXAMPLE suppose θ is in the II quadrant, i.e. $\theta = \pi - \theta_0$.

then $\sin\theta = y = \sin\theta_0$
$\cos\theta = -x = -\cos\theta_0$

THE SAME ARGUMENT CAN BE USED TO GET SIGN TRIG IDENTITIES:



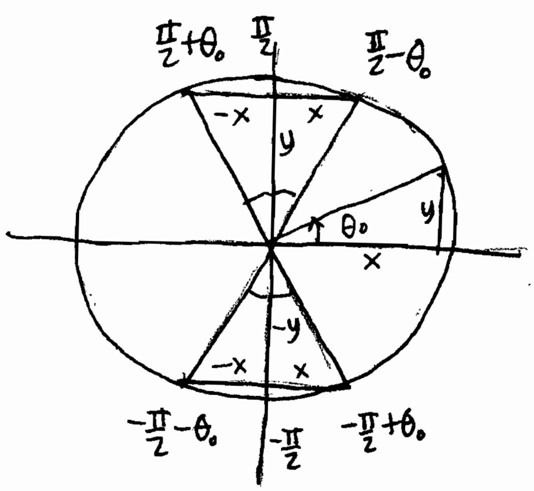
Pretend θ is 30° to draw picture.
Locate π−θ.
Then from diagram

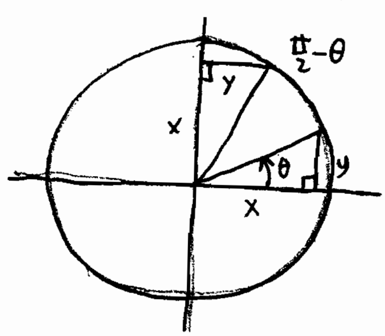
$\sin(\pi-\theta) = \sin\theta$
$\cos(\pi-\theta) = -\cos\theta$.

OR for −θ: $\sin(-\theta) = -\sin\theta$ (odd)
$\cos(-\theta) = \cos\theta$ (even)

(Try it for $\pi+\theta$)

The interchange identities come from hanging the reference triangles off the y-axis.





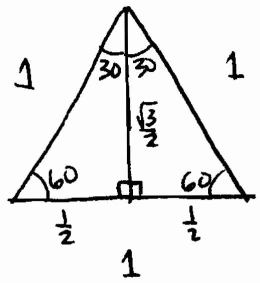
For example, pretend θ is 30°:

$\cos(\frac{\pi}{2}-\theta) = \sin\frac{\pi}{2}$

$\sin(\frac{\pi}{2}-\theta) = \cos\frac{\pi}{2}$

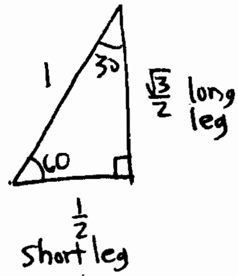
(Try for $\frac{\pi}{2}+\theta$)

TRIG: SPECIAL ANGLES

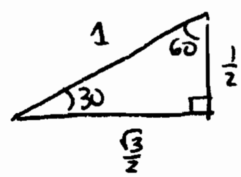


equilateral triangle

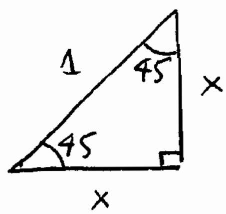
"STANDING UP"



"LYING DOWN"



(Note $\frac{\sqrt{3}}{2} = \sqrt{1-\left(\frac{1}{2}\right)^2}$ pythag. theorem)



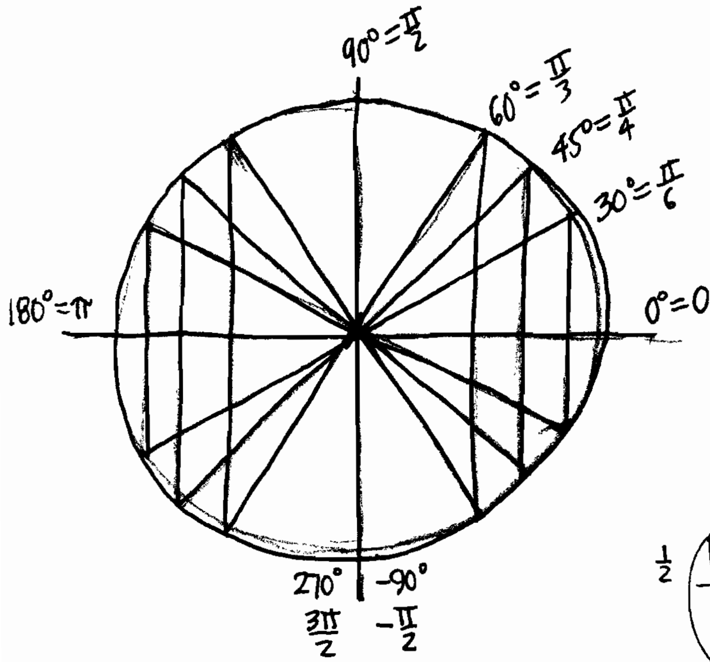
$2x^2 = 1$

$x = \frac{1}{\sqrt{2}}$

isosceles right triangle

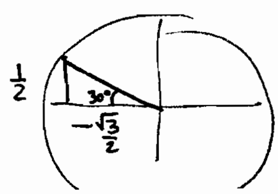
These reference triangles give you the trig functions of $30°, 45°, 60°$ or $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$.

(Recall $\frac{\theta_{rad}}{2\pi} = \frac{\theta_{deg}}{360}$ or $\theta_{rad} = \frac{\pi}{180}\theta_{deg}$)



Putting reference triangles in all 4 quadrants gives us (together with the axes) 16 different angles all of whose trig functions can be calculated from the special angles we know.

EX $\cos\frac{5\pi}{6} = \cos\left(\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$



angles in argument of trig functions in calculus are ALWAYS radians unless explicitly indicated to be degrees as in $\cos 30° \neq \cos 30$.
Degrees are just a tradition and have no mathematical origin — we use them only for convenience to visualize angles.

on a nonunit circle, length quantities scale by the radius r:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{aligned} r &= \sqrt{x^2+y^2} \\ \tan\theta &= \frac{y}{x} \end{aligned}$$

relations between cartesian & polar coords.