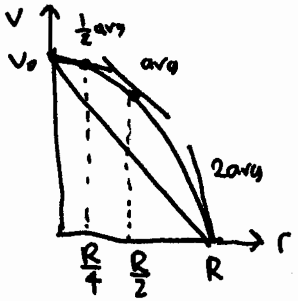
Blood flow model (Stewart 3.3)

$$V = \frac{P}{4\eta l}(R^2 - r^2) = \underbrace{\frac{PR^2}{4\eta l}}_{V_0}\left(1-\frac{r^2}{R^2}\right) = V_0\left(1-\frac{r^2}{R^2}\right)$$



velocity gradient: $\frac{dV}{dr} = V_0\left(0 - \frac{2r}{R^2}\right) = -2\frac{V_0 r}{R^2} = 2\left(\frac{r}{R}\right)\left(\frac{-V_0}{R}\right)$

$$\left.\frac{\Delta V}{\Delta r}\right|_{\substack{\text{from } r=0\\ \text{to } r=R}} = \frac{0-V_0}{R-0} = -\frac{V_0}{R} \leftarrow \text{average velocity gradient}$$

Conclusion: the velocity gradient is twice the fractional radius $\left(\frac{r}{R}\right)$ times the average velocity gradient from the center to the outside edge of the vessel.

If $P = 3000$ dynes, $\eta = 0.27$, $l = 3$ cm, $R = .01$ cm, (3.3.27)

then $V_0 = 0.926$ cm/sec

$$-\frac{V_0}{R} = -\frac{0.926 \text{ cm/sec}}{.01 \text{ cm}} = -\frac{0.0926 \text{ cm/sec}}{.001 \text{ cm}} \leftarrow$$

10 tickmarks from $r=0$ to $r = R = .010$

on the average the velocity decreases by .0.0926 cm/sec as the radius increases by 1/10 the fractional distance from the center to the edge.

Note when $\frac{r}{R} = \frac{1}{2}$, then $\frac{dV}{dr} = 2\left(\frac{1}{2}\right)\left(-\frac{V_0}{R}\right) = -\frac{V_0}{R}$ = average gradient

so halfway to the edge the slope of the tangent line in the above diagram is the same as the overall secant line, namely the gradient at the halfway point equals the average gradient.

when $\frac{r}{R} = 1$ we get twice the average gradient.