ratio limits

\[
\lim_{x \to a} \frac{f(x)}{g(x)} \quad \text{separately evaluate}\ 
\text{ top & bot values or limits}\]

\[\frac{C_1}{C_2} \to \frac{0}{0}, \quad \frac{C_1}{0}, \quad \frac{0}{C_2}, \quad \frac{0}{0}\]

4 cases:

\[
\text{limit = value (no problem)}
\]

"reasoning" for \(\frac{C_1}{0} \to 0\) limits

let \(0^+ = \text{abbreviation for approaching 0 thru positive values}\)

let \(0^- = \text{abbreviation for approaching 0 thru negative values}\)

\[
\frac{C_1}{0^+} = \text{sgn}(C_1) \times \infty \quad \text{as } x \to a, \ g(x) \to 0 \text{ thru positive values, so overall sign is that of } C_1
\]

\[
\frac{C_1}{0^-} = -\text{sgn}(C_1) \times \infty \quad \text{as } x \to a, \ g(x) \to 0 \text{ thru negative values, so overall sign is opposite to that of } C_1
\]

evaluation for \(\frac{0}{0}\) limits

Must do algebra manipulation to remove problem if possible, reducing limit to one of the previous cases by:

1) factor, cancel, continue
2) expand (multiply out), cancel then
3) if difference of sorts, multiply by "conjugate" sum (top & bot), combine, cancel, continue
4) if \exp, \ln, \text{ trig functions involved, use special identities to manipulate expression,}

ALL OF THIS ALSO APPLIES TO LIMITS AS \(x \to \pm \infty\).

Arithmetic with infinity

some symbolic arithmetic with \(\infty\) (treating it like a number)
is possible with \(\infty\) (understood to really mean positive \(\infty\) here), some is not

OKAY: \(\infty \cdot \infty = \infty\), \(\infty + \infty = \infty\), \(\frac{\infty}{\infty} = 0\)

\[e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0, \quad \ln 0^+ = -\infty, \quad e^\infty = \infty\]

unambiguous limits

\(\ln \infty = \infty\)

NOT OKAY: \(\frac{\infty}{\infty} \neq 1\), \(\infty - \infty \neq 0\) ambiguous limits, due to competition: any outcome is possible